

FIELDS NOTES

VOLUME 15:2

THE FIELDS INSTITUTE FOR RESEARCH IN MATHEMATICAL SCIENCES

Article on
**Delay Equations
in Electrodynamics**

by D. -A. Deckert & G. Hinrichs
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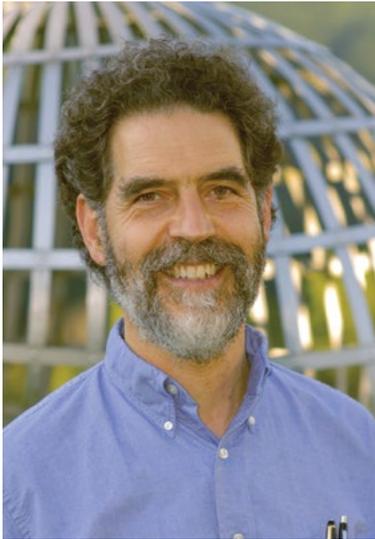
BIG DATA

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Fields Institute** pg 9

Message from the Director



As you may know, I am stepping down for health reasons and will be returning to my faculty position at McMaster. It is now my pleasure to announce, if you have not heard, that the Director search was successful and the next Fields Institute Director will be Ian Hambleton, who began his tenure here on July 1, 2015.

Ian is the Britton Professor of Mathematics at McMaster University. He is a well known topologist with a doctorate from Yale University and has been a leading researcher in the Canadian

mathematical community. He will be taking the position as an able and experienced administrator. The Director Search Committee worked very successfully, and it is a testament to the success of the Institute that there were so many excellent and qualified candidates that were considered in the competition for the position. I'd like to thank this committee for their work, and also I would like to thank the numerous candidates who made themselves available for consideration.

Scientific Programs

It is a great inspiration to me to have the Institute full of activity, to be surrounded by bright researchers who are actively engaged in collaborations, in a broad spectrum of areas of the mathematical sciences. It has kept the Institute staff particularly busy recently, as we are just coming through a period of three simultaneous thematic and focus programs, with a typical week also involving some sort of additional general scientific activity such as a stand-alone workshop or a period of in-house collaboration with industry. I would like to give you a summary of what programs we have held this year, and the ones that are coming up on the horizon.

The Thematic Program on Statistical Inference, Learning, and Models for Big Data took place from January to June 2015, with a very successful series of six principal workshops or conferences, along with a full spectrum of allied activities held at Fields as well as elsewhere in Canada. The organizing committee consisted of Nancy Reid (Toronto), Yoshua Bengio (Montréal), Hugh Chipman (Acadia), Sallie Keller (Virginia Tech), Lisa Lix (Manitoba), Richard Lockhart (Simon Fraser), and Ruslan Salakhutdinov (Toronto). The principles of this program were to delve deeply into very current theoretical areas of statistics, those that have had a large recent impact and been a driving force in the current focus on problems of big data. But as well to look broadly at the growing number of important applications of the statistical sciences to other areas of science and the social sciences.

We also ran two shorter programs this semester. Firstly, we take note that it is now 2015, a century after the revolutionary 1915 paper of Einstein formulating the general theory of relativity; in this light we hosted the *Focus Program on 100 Years of General Relativity*, organized by Spyros Alexakis (Toronto), Mihalis Dafermos (Princeton), Luis Lehner (Perimeter Institute and Guelph), Harald Pfeiffer (CITA), and Eric Poisson (Guelph). It is also a time at which the rigorous mathematical foundations of Einstein's equations are receiving attention, and where the mathematics and the physics communities are increasingly engaged in collaborations and exchanges of ideas. Some of the highlights of this two-month program include the very well attended public lecture of Clifford Will (Florida) and the *International Conference on Black Holes*, which was held at Fields this June. A second short *Thematic Program on Delay Differential Equations* took place in May this year, which featured themes for each of its four weeks of activity and a Distinguished Lecture series by John Mallet-Paret (Brown University). Its organizing committee consisted of Odo Diekmann (Utrecht), Sue Ann Campbell (Waterloo), Stephen Gourley (Surrey), Yuliya Kyrchko (Sussex), Eckehard Schöll (TU Berlin), Michael Mackey (McGill), Hans-Otto Walther (Giessen), Glenn Webb (Vanderbilt), and Jianhong Wu (York).

Coming up in the future are a number of thematic programs in very interesting and currently active areas of research, and spread broadly across the mathematical sciences. Starting already this July we are having a *Thematic Program on Computer Algebra*,

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Cover: Barrett Lyon / The Opte Project "The Internet in 2015" Permission to reproduce granted from Barrett Lyon. Image sourced from <http://www.opte.org/>

Opte Project was originally created to generate a picture (or map) of the Internet. The visualization is a collection of programs that collectively output an image of every relationship of every network on the Internet.

which will feature workshops on a variety of aspects of the mathematics of making computers do algebra and integrals and solve differential equations. Stephen Watt (Waterloo) is a lead organizer of this program, along with Erich Kaltofen (North Carolina State University), George Labahn (Waterloo), Peter Paule (Johannes Kepler University, Linz), Marie-Françoise Roy (Rennes), Nikolay Vasilyev (Steklov Institute, St. Petersburg), and Lihong Zhi (Chinese Academy of Sciences). A Coxeter Lecture Series by Victor Schoup (Courant Institute) is planned.

Coming up for the winter term 2015-2016 is a *Thematic Program on Multiscale Scientific Computation: from Quantum Physics and Chemistry to Material Science and Fluid Mechanics*. The two lead organizers are Ionut Danaila (Rouen, France) and Bartosz Protas (McMaster), and they are joined by Weizhu Bao (Singapore), Qiang Du (Columbia), Nicholas Kevlahan (McMaster), Yvon Maday (Paris 6), Michael Siegel (New Jersey Institute of Technology) and Lennaert van Veen (UOIT). The program will address three themes, built around advanced computing and theoretical fluid mechanics, computation of quantum systems in cold-matter physics and chemistry, and multiscale modeling and its applications: from weather and climate models to models of material defects. The organizers plan for a Coxeter Lecture Series, most likely to be given by Bjorn Engquist (University of Texas - Austin).

The fall term 2016-2017 has been set for a Thematic Program on Combinatorial Algebraic Geometry, organized by David Cox (Amherst College), Megumi Harada (McMaster), Diane Maclagan (Warwick), Gregory Smith (Queen's), and Ravi Vakil (Stanford). They are planning to have three principal workshops, one introductory and then one on Convexity in Algebraic Geometry and a third one on Combinatorial Moduli Spaces and Intersection Theory. A Coxeter Lecture Series by Andrei Okounkov (Columbia, Fields Medal 2006) is planned for the semester.

Centres Initiative

The Fields Institute has a long history and track record of building and supporting activities that bring together mathematical scientists and researchers in other areas. Examples include our series of seminars and workshops in quantitative finance, our seminar series on industrial optimization, the activities of the Centre for Mathematical Medicine, and the various events and programs that we support in mathematics education, in particular the Math Ed Forum and Math Circles. These activities fall naturally into subsets identified by the similar interests and goals of the participants. We felt, however, that these were being handled non-uniformly by the Institute, and therefore we came up with a plan for establishing Centres as organizational principles with uniform governance structures and well defined contributions from Fields. Our vision of a Centre is an umbrella organization that will provide and identifying unity and visibility to a specific area in the mathematical sciences. Typically, but not exclusively, the subject area is an interdisciplinary one, and the Centre serves the purpose of bringing together expertise cutting across departmental lines, as well as academic, industrial, and governmental divides. A proposal has been approved for our first centre under the auspices of this initiative, which is the Centre for Financial Industries (CFI), and we should start to see CFI activities starting up in the coming year. We are expecting to have proposals for centres from other activity groups in the near future, and I could imagine one for industrial mathematics (such as industrial optimization), for mathematics education, and for a reformulation of the Centre for Mathematical Medicine.

New affiliates

The Institute is very pleased to have welcomed two new Partners during the year, both on the corporate side of our affiliation program. This sort of association gives the affiliate a seat on the Fields Institute's Industrial Advisory Board, a representative on the Fields Institute Corporation, as well as the ability to host and participate in certain events at the Institute. The two new Corporate Affiliates are Waterfront International Ltd., a quantitative finance research firm based in Toronto, and Maplesoft, the well known provider of high-performance software tools for engineering, science, and mathematics. It is well known that the computer algebra program Maple has its origins in Waterloo, so this partnership is a very natural one. Further to this arrangement, Maplesoft will be contributing both intellectually and with a sponsorship to the Thematic Program on Computer Algebra that is starting up at Fields this July.

There was some sad news for us this past year, in that James Stewart passed away on December 3, 2014. A former McMaster colleague and well-known calculus textbook author, he was a long time friend and benefactor of the Fields Institute. The Library seminar room at the Institute bears his name, and he contributed both money and the hospitality of his home for the Fields Medal Symposia. We will miss his presence at our events. In his will he has remembered the Institute in terms of a portion of his estate, for which we are very grateful. We are thinking of using this contribution to establish the kernel of an unrestricted endowment, with the plans to normally use a portion of the proceeds to continue to support the Fields Medal Symposia into the future. We are planning to name him in some special way (yet to be formulated) as one of the founding and contributors-in-perpetuity to the symposia.

Staff

The staff at the Fields Institute are wonderful, very generous with their time, extremely talented, active, and considerate. The Institute would not do what it is able to do without such dedication. We have a complement of 14 staff members now, of whom seven have come to Fields in the past two years.

What I can say is that the staff at the Institute are its heart and soul. With a view to our resources compared to the level of our activities and the numbers of participants, I like to say we do twice as much with half the resources. This is a factor of four difference, of course. They are famous around the world in the international community, and I am justifiably proud of their contributions.

Respectfully submitted,

Walter Craig, Director
June 25, 2015

Maplesoft Becomes Corporate Affiliate

Maplesoft and the Fields Institute Partner to Enhance Use of Technology in Mathematical Exploration

IN JUNE OF THIS YEAR MAPLESOFT and the Fields Institute for Research in Mathematical Sciences entered into a partnership that is aimed at promoting the study and use of math in education, research and industry.

As part of the partnership, Maplesoft will become a Corporate Affiliate of the Fields Institute, and donate the latest version of its mathematical computation tool, Maple 2015™, to outfit every computer on the Institute's grounds. Maplesoft will also collaborate on events and workshops which will examine the use of the internet, computational tools and other technology and tools to enhance the quality of math education.

Maplesoft has a long history of partnering with universities, research institutions and societies that encourage the study and use of math. From the time of the company's inception, its tools have been used to help students better understand math concepts, assist instructors in the classroom, and enable researchers to accelerate their work. Maplesoft technology is also used in industry where applying mathematical techniques to engineering concepts helps fast track development and brings products to market quicker.

"Maplesoft has a strong relationship with the Fields Institute, and we have a shared vision for the future," said Jim Cooper, President and CEO, Maplesoft. "In extending our partnership, we see mutual benefits for both organizations in terms of pooled resources, knowledge and talent. Maplesoft is delighted to join with the Fields Institute to support and enhance mathematical activity around the world."

The Fields Institute sees that its role as a centre for mathematical research activity will be bolstered by this partnership.

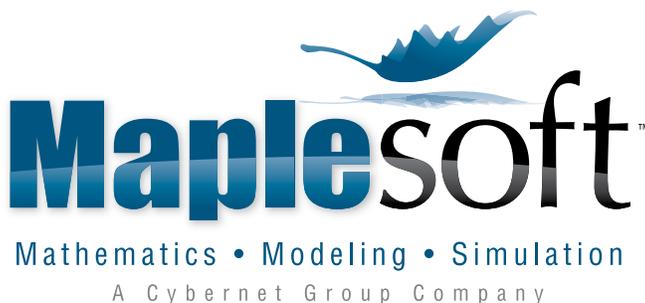
"We welcome Maplesoft to the ranks of select mathematically sophisticated companies who make up our Affiliates group," said then Director of the Fields Institute, Walter Craig.

"We share many common interests including the pursuit of mathematical knowledge and exploration. As a pioneer in providing tools for e-learning, Maplesoft's contribution will prove very valuable to Field's own initiatives in nurturing the online instruction of mathematics."

This agreement is part of the Corporate Affiliate Program offered by the Fields Institute. All activities at Fields are open to the industrial

and commercial communities; an agreement between a Corporate Affiliate and the Institute, however, establishes a particular relationship of great mutual benefit. Affiliate Members receive multiple benefits such as access to activities, the ability to propose programs (to be hosted at Fields), recruitment opportunities, as well as involvement in the governance of the Institute itself.

For more information about how to become a Corporate Affiliate visit the Fields website, or download the PDF available here: <http://www.fields.utoronto.ca/aboutus/CorporateBenefits2015.pdf>. You may also contact the Institute directly at (416) 348-9710 x 3024 or by email at inquiries@fields.utoronto.ca, and the Communications team will field any questions you may have.



About Maplesoft

Maplesoft™ is a subsidiary of Cybernet Systems Co., Ltd. in Japan. It is a leading provider of high-performance software tools for engineering, science, and mathematics. Its product suite is designed to reflect the philosophy "that given great tools, people can do great things."

Maplesoft's core technologies include the world's most advanced symbolic computation engine and revolutionary physical modelling techniques. Combined together, these technologies enable the creation of cutting-edge tools for design, modelling, and high-performance simulation. Engineers, scientists, and mathematicians use Maplesoft products to enable them to work better, faster, and smarter.

Maplesoft's customers include Ford, BMW, Bosch, Boeing, NASA, Canadian Space Agency, Canon, Motorola, Microsoft Research, Bloomberg, and DreamWorks, covering sectors such as automotive, aerospace, electronics, defence, energy, financial services, consumer products, and entertainment. Over 90 per cent of advanced research institutions and universities worldwide, including MIT, Stanford, Oxford, the NASA Jet Propulsion Laboratory, and the U.S. Department of Energy, have adopted Maplesoft solutions to enhance their education and research activities. Visit www.maplesoft.com to learn more.

*The Editors of FieldsNotes,
with files from Maplesoft*

Partnership with SpringerOpen

The Fields Institute has announced a new partnership with Springer, specifically SpringerOpen, in order to bring out its publications Mathematics-in-Industry Case Studies (MICS) and Fields Mathematics Education Journal (FMEJ). These publications will be open access, with the Institute covering the charges for article-processing.

Prior to this the journals had been published and hosted by the Institute itself. With SpringerOpen, it is the hope of both parties to increase the visibility and the ease of access for both academics and researchers to the high level activity that is occurring at Fields.

Since 2008, MICS has striven to meet the publication needs of the burgeoning community of mathematicians who work on problems relevant to industry and its development. Huaxiong Huang, (York and Fields), and Chris J. Budd (Bath) act as the current co-editors-in-chief.

Founded last year, the FMEJ has highlighted research that has particular interest for the mathematics education community. The journal's editors-in-chief are Donna Kotsopoulos (Wilfrid Laurier), and Dragana Martinovic (Windsor).

Both publications are supported by a long list of internationally renowned researchers as part of their respective editorial boards.

Fields feels that by allowing open access to the results of the activities that Fields sponsors, their aim to provide access and visibility will be better met as the Institute continues to grow its programming.

Walter Craig, the outgoing Director, and Matheus Grasselli, current Deputy Director of the Fields Institute, said in a joint statement that “these two journals reflect the fact that both mathematics education and industrial mathematics have been core areas of strength at the Fields Institute for many years. We are also committed to continue to play a leadership role in the open access format, which we believe is the future of academic publishing, and are happy to extend our existing partnership with Springer into this domain.”

“Springer is pleased to welcome the Fields Mathematics Education Journal and the Mathematics-in-Industry Case Studies Journal to SpringerOpen, expanding further our co-publishing relationship



with the Fields Institute for Research in Mathematical Sciences. These journals are guided by motivated editors-in-chief and editorial boards and we look forward to seeing the journals flourish in the next years,” said Elizabeth Loew, Executive Editor, Mathematics, Springer.

About Springer

Springer Science+Business Media is a leading global scientific, technical and medical publisher, providing researchers in academia, scientific institutions and corporate R&D departments with quality content via innovative information products and services. Springer is also a trusted local-language publisher in Europe – especially in Germany and the Netherlands – primarily for physicians and professionals working in healthcare and road safety education. Springer published roughly 2,400 English-language journals and more than 9,000 new books in 2014, and the group is home to the world’s largest STM eBook collection, as well as the most comprehensive portfolio of open access journals. In 2014, Springer Science+Business Media generated sales of approximately EUR 959 million. The group employs some 8,500 individuals across the globe.

*The Editors of FieldsNotes,
with files and photos from Springer Science+Business Media*

Caribou Math Contests:

Not just a contest but a way of learning and enjoying math



LAST ISSUE WE DISCUSSED how math can be made fun in order to make it a more enjoyable subject for students. Recent news reports found that many teachers struggle with just trying to get through the lesson plans, and often make any enjoyment of the subject take a backseat.

Thomas Wolf of Brock University thinks that learning and using math does not need to be dry, and to prove this he has developed Caribou Math Contests. Dr. Wolf has divided these contests

into six separate events spread across a school year, and each of these into five different grade categories: grades 3-4, 5-6, 7-8, 9-10, and 11-12.

“What is special with Caribou Contests is that they are held online which allows us to post all results and statistics on the evening of the contest day. The contests feature interactive questions as well as multiple choice questions, and finally they require minimal effort to be administered by schools,” says Wolf.

The web portal for the contest also showcases all previous contests as online practice tests. This allows anyone to attempt them and then see which rank they would have obtained in the real contest. They are also used as a teaching tool within the class room together with video solutions. In order to ensure that all students have access to these contests, Wolf has maintained that all contests for the youngest age group are to remain free of charge, and in addition for all other grades the first contest each school year is also free so that students and teachers can decide whether they want to sign up for the rest of the school year. A yearlong sign up for 6 contest days costs less than what other contests charge for a single contest.

“We have a relatively wide spectrum of easy and hard questions. All students can be successful in some questions and for even the best students some problems are really challenging,” he says.

The variations in difficulty serve a number of purposes within the test questions. The primary goal is to make sure that all ranges of ability and instruction can answer a good portion of the questions.

“Most questions can be done in a few seconds if one has a good idea so the better part of the 60 minutes are available for the few harder questions,” Wolf explains.

“Also, for having a ranking distinguishing between 3000 participants, one needs more than a few hard questions. But more importantly, what we want to achieve is to introduce many students to the excitement and fun of thinking about mathematical problems. The most time our participants think about our mathematical problems is not the time during the contest but the time spent on our practise tests and video solutions before and after the contest as one can see

from our online statistics of over 400,000 practice tests taken since 2009.”

The actual learning of how to solve questions is supported by a Caribou Contest YouTube channel. In the clips a volunteer student (Oliver Wolf) explains some of the complicated and fun math behind some of the math questions. Part of the interesting thing to watch is that it has been the same volunteer since the channel start in September 2011. With three years under his belt, you can see him grow from an elementary student into a high schooler, with many interesting haircuts along the way.

The contests are not only innovative in the openness in which they are administered, but in some of the technology and puzzles that were developed specifically for them. One of the puzzles that Wolf has developed he calls a “calcrostic problem”.

These puzzles are similar to a Sudoku or magic square puzzle where a value entered into a “box” must interact in specific way to the surrounding values.

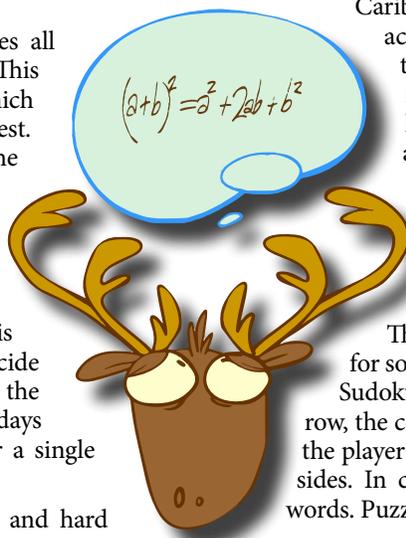
“The name is derived from CALculational aCROSTIC problem, - it’s a word not known to Google before,” explains Wolf with a bit of pride.

Caribou boasts a number of math based games that can be accessed at any time, and have been over two-million times since 2011. These classic games are used as interactive questions in contests and are especially liked by students. They boost logical thinking skills and also the students confidence.

Mathematical Developments

Calcrostics

What is it that makes some puzzles addictive, like Sudoku, Rubik’s Cube or cross word puzzles? They have a high branching factor of possible steps for solution but when completed many rewards follow: In Sudoku each found digit satisfies three requirements (the row, the column and block), when Rubik’s Cube is completed the player is rewarded 6 times when looking at all uni-colour sides. In crossword puzzles a letter typically completes two words. Puzzles, like:



EDKH	÷	KF	=	AA
-		+		+
EDB	x	J	=	EHCG
=		=		=
EEJD	-	DK	=	EEAE

where each letter stands for a digit give 6 rewards, 3 horizontal and 3 vertical correct computations.

In an attempt to improve such puzzles Wolf created similar ones which give even more reward when being solved with two more diagonal conditions, like:

$$\begin{array}{rcccccc}
 AB & + & AB & = & BC \\
 - & \times & - & \div & + \\
 DE & \times & E & = & FE \\
 = & = & = & = & = \\
 C & \times & DC & = & DBB
 \end{array}$$

Furthermore it was possible to create a scheme which allows to create instantly infinitely many such puzzles.

Since January 2010 their web page has a daily new calcroscopic problem and the solution of previous day's problem. Also the North East Asian Mathematics Competition held March 2015 in Nanjing, Jiangsu, China, with participating high school students from nine Asian countries had, as part of their event, one contest that exclusively featured calcroscopic problems from Caribou.

Correlation of Solutions

To ensure that Caribou contests are fair it is interesting to study unlikely correlations between answers of different students from the same school. A first version of their correlation test was based on the assumption that all questions are equally hard and all wrong options are equally often selected. A planned newer version takes into consideration how often each question was solved correctly and how often each of the wrong options was chosen, both depending on the strength of the students. This generalization is possible due to the large number of participants in each contest.

Arbitrary Size Sudoku

The Caribou interactive games, like Mastermind, Sudoku and Nim are freely available online for practise. Most have an unlimited number of initial positions to play against. The usual 3x3 Sudoku giving a (3x3)x(3x3) square can already be challenging. What Wolf developed is a method to generate a large class of arbitrary size Sudoku. The generation of arbitrarily large and difficult Sudoku is possible in a time only growing linearly with the area of the puzzle. What does take longer is to generate Sudoku that are minimal in the sense that taking away any initial number would give up the property of a unique solution.



Presented with "Caribou Cups" - top student in each grade
 Back Row: Ian Kennedy & Cory McLean
 Front Row: Dillon McLean Gr. 4, Isaac Puar Gr. 5, Reese Tam Gr. 3, Collingwood School, West Vancouver, BC

Colouring Maps

A game to colour maps with a minimal number of colours is already available online by a different publisher. But that game has a fixed number of initial maps to colour. The Caribou map colouring program is able to generate an arbitrary number of initial maps with different connectivity properties and arbitrary size giving a rich source to study solution strategies.

Coming Soon

New interactive questions to be available in the coming school year include interactive geometry constructions, map colouring, the Hackenbush game, Tangram and the Sokoban game.

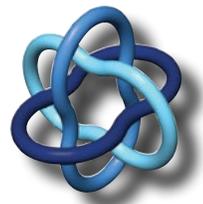
All contest information, and how to participate, is available on the Caribou home page cariboutests.com.

Caribou Contests is grateful to the Fields Institute for the support received over the previous years.

Thomas Wolf has provided the calcroscopic puzzle below where for each letter a digit needs to be found such that the value of all 7 rows, 7 columns, 11 diagonals from top left to bottom right and 11 diagonals from top right to bottom left are all zero. The solution will be available in the next issue!

Thomas Wolf (Brock) and the Editors of Fields Notes

$$\begin{array}{cccccccccccc}
 abe & - & j & + & - & hcbhgb & - & hf & + & hb & - & hb & - & - & hchbef \\
 - & - & + & - & + & + & + & + & \times & + & - & \times & - & - & - \\
 j & - & j & + & hf & - & hj & - & b & + & hi & - & hb & - & hb \\
 - & + & - & - & + & + & + & - & + & + & + & + & + & + & + \\
 hcbhge & - & j & \times & i & - & d & + & - & ia & - & hcbfde & + & e & - \\
 - & + & + & - & - & + & - & + & + & - & + & - & - & - & - \\
 - & he & - & cja & - & c & + & c & + & f & - & - & hchgjc & - & hcbfge \\
 - & - & - & + & - & + & + & + & - & + & - & - & - & - & + \\
 dbd & + & - & becidh & + & - & cga & - & hah & + & bid & - & bfh & - & - & becihb \\
 + & + & - & + & - & - & + & + & + & - & + & + & - & + & \div \\
 becijc & + & - & ccg & - & - & hchae & - & hchedb & + & - & e & - & bejfhe & + & bfb \\
 + & - & - & - & - & - & + & + & + & + & - & - & + & - & + \\
 - & hcbfjf & - & becijc & - & hii & - & - & hchiea & - & bfj & - & bfb & + & bejeji
 \end{array}$$



FIELDS MEDALS

IN HONOUR OF STANISLAV SMIRNOV



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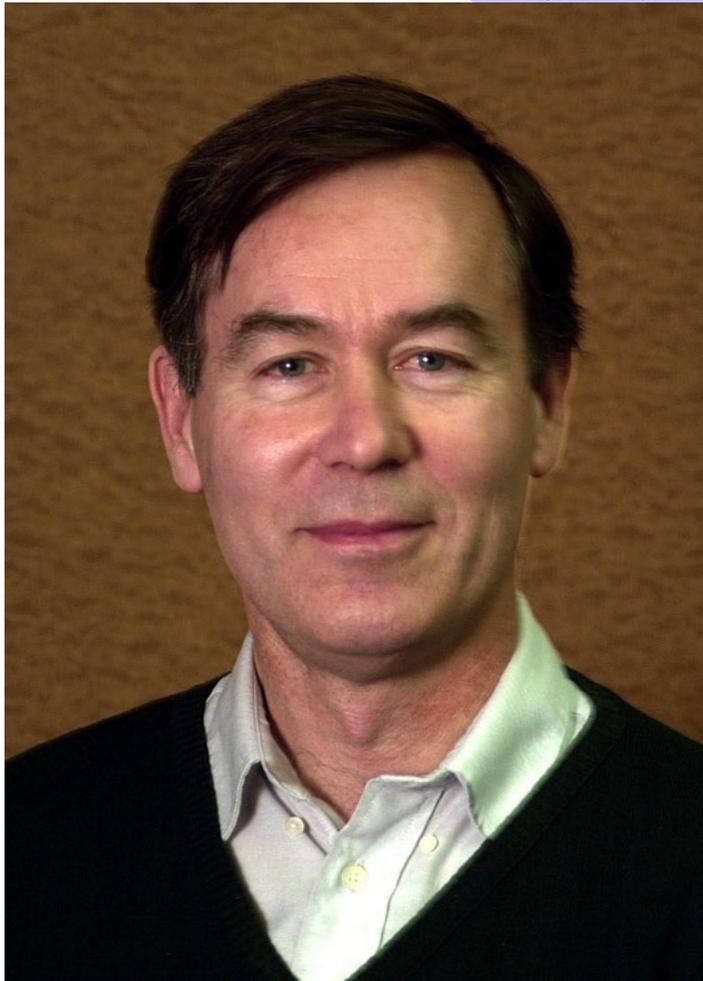
Q & A: Ian Hambleton

The new Director of the Fields Institute

In this issue we have the pleasure of introducing the Fields Institute's new Director, Dr. Ian Hambleton.

Dr. Hambleton sat down with us to answer some questions and to tell us a little about himself.

BORN IN TORONTO in 1946, he was brought up in a household that put a high value on music, books and learning. He graduated from the University of Toronto with a B.Sc (1968) in Mathematics and Physics, and an M.Sc (1969) in Mathematics. During his undergraduate years he took several inspirational courses from the internationally renowned geometer, Professor H. S. M. Coxeter, which had a decisive influence on his future career. Dr. Hambleton went on to receive his doctorate from Yale University in 1973. From 1973-1975 he was a L. E. Dickson Instructor at the University of Chicago. Later he joined the faculty at McMaster and becoming a full professor in 1984. While at McMaster, he served as Chair of the Department of Mathematics and Statistics for three terms, was several times elected to the Senate and Board of Governors, and served as the President of the McMaster Faculty Association. He is a prominent Canadian mathematician whose research in geometry and



Fields Institute's new Director, Dr. Ian Hambleton

topology connects a broad range of mathematics.

Joining the Fields Institute is Dr. Hambleton's newest adventure, following the Directorship of Walter Craig, who had to step down from the position in June for health reasons.

Dr. Hambleton says that his appointment to Fields represents an opportunity to build on its established strengths and to develop new directions.

"There is a lot to learn" he says. "Mathematics can be, and now often is, about entrepreneurship, start-ups, finance, design and engineering. Fields has been already doing a lot of this and it is something I feel very excited about."

Dr. Hambleton points out that the Fields Institute is seen as a top level institute with a high international and national reputation. He indicates that every effort will be made to reinforce and enhance its standing. "The Fields Institute belongs to an elite group of mathematical research institutions – like MSRI in Berkeley and the Isaac Newton Institute in Cambridge.

One of the activities that distinguishes us is the annual Fields Medal Symposium," explains Dr. Hambleton. "Joining the Fields Institute at this time I get to be involved with the 4th Annual Symposium, to honour of the work of Stanislav Smirnov. This is exciting because it brings together all types of academics, students and those interested in the sciences under our roof in order to talk about mathematics. Each year we focus on a recent winner of the Fields Medal, and their public talks are always a high point in the celebration."

The Fields Medal Symposium has been celebrated annually for the last four years. This year's Symposium will be held from October 19-22, 2015

The numerous public events and outreach activities at Fields continue to build links for mathematicians to the broader academic and industry communities.

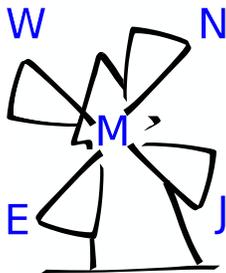
"The Fields Institute's key role in promoting fundamental research, international collaboration, and connecting mathematics to the complexities of the 'real world' is more important than ever.

"We would like to open up people's eyes to the many opportunities there are in mathematics."

The Editors of FieldsNotes

Fields supports new North-Western European Journal of Mathematics

European mathematical research labs are leading the way in open access journals



N THIS JUNE, A COLLABORATION of academic institutions across Europe and North America announced the launch of a new open access research journal: the North-Western European Journal of Mathematics (NWEJM), based at the Université Lille 1, a Fields Institute Affiliate University.

E The founders of NWEJM have created a publication that will allow researchers and students access to ground-breaking mathematical research without being restricted by pay walls and publication fees. Walter Craig, the former Director of Fields, will serve on the Editorial Board of the journal in order to represent the interests of the Institute.

The journal accepts original papers from all areas of pure and applied mathematics, as well as those exploring the history of mathematics.

In a letter provided to the Fields Institute promoting the launch, Editor-in-Chief Serge Nicaise and Managing Editor Nicolas Wicker say that the aim of the NWEJM is to “set up a high quality open access journal with academic support.”

“The time is proper to start such a project as libraries are having difficulties [meeting] the [costs] of the major publishers and Europe is putting researchers under pressure to submit papers to open access journals,” they wrote, noting the two impetuses that had brought about the creation of the publication.

NWEJM uses the diamond model of open access publishing. This is a relatively new concept that ensures neither authors nor readers are saddled with fees for publication or access. Other open access models offset their costs by requiring a fee from authors, or by providing guidance in securing specific funding support through other institutions.

Throughout the submission process researchers will retain

copyright to their work. Again, no charges will be levied on authors, and articles will be free to access after they have been accepted. Hard copies of complete volumes will also be made available at cost (shipping and printing), while online access will always be free and unhindered.

NWEJM’s publication is supported by the Department of Mathematics of Lille 1, the French Mathematical Society, the Dutch Mathematical Society, the Luxembourg Mathematical Society, and the Fields Institute. Access to the journal has been organized through a non-profit publisher. This combination of efforts will ensure that this journal will remain open access.

The move to open access has been encouraged by both European and Canadian Governments over the last few years. In Canada, NCERC and other agencies have requested that results of scientific research and programs that they have directly supported be made available as open access publications. Their desire matches Fields’ long standing wish to make the research done at the Institute accessible to all.

The NWEJM model is one of the many that the Fields Institute is examining.

The first volume of the North-Western European Journal of Mathematics was made available in May 2015, and can be found here:

<http://math.univ-lille1.fr/~nwejm/#Current>

Information on becoming an Affiliate University for the Fields Institute can be found here:

<http://www.fields.utoronto.ca/aboutus/AffiliateBenefits2014.pdf>

*The Editors of FieldsNotes
with files from the North-Western European Journal of Mathematics*

Workshop on Representation Theory & Analysis on Lie Groups over Local Fields

May 7-10, 2015 - University of Ottawa



DURING A SUNNY WEEKEND this May, the University of Ottawa proudly hosted the *Workshop on Representation Theory and Analysis on Lie Groups over Local Fields*. This hugely successful conference drew almost 60 participants from all over Canada, the US, Mexico, Europe and Israel. These included close to 30 graduate students and new PhDs, a majority of whom were supported by funding granted by the Fields Institute. Junior researchers were invited to apply to give a talk in the months leading up to the workshop. Of the thirteen abstracts submitted, seven were chosen and the excellent, enthusiastic talks given by these junior researchers greatly contributed to the cutting-edge nature of the conference.

A key draw to this workshop was the pair of mini-courses which intertwined the theory of spherical varieties from analytic and geometric perspectives across both real and p -adic fields. Bernhard Krötz (Paderborn) delivered a sequence of three lectures for his mini-course, entitled *Volume growth on homogeneous spaces, Real spherical spaces I: Geometry and Real spherical spaces II: Harmonic Analysis*. Yiannis Sakellaridis (Rutgers), gave a complementary course of three lectures, covering *Geometry of toric and spherical varieties, and the theory of asymptotics*, as well as *Plancherel decomposition and scattering theory*. Krötz and Sakellaridis were dynamic and the audience participation was strong; their lectures really set the stage each morning for a fruitful day.

With a total of 15 conference talks on top of the two mini-courses over three-and-a-half days, attendees were pleasantly exhausted and fulfilled by Sunday afternoon. The talks covered a broad range of problems and directions in representation theory, but spherical varieties and unitary representations were central themes to almost all of them. The lectures by Moshe Adrian (Toronto), Ioan Badulescu (Montpellier), Max Gurevich (Technion), and Paul

Mezo (Carleton) were on the back-and-forth relation between the Langlands program and harmonic analysis, while Julia Gordon (UBC) spoke about the calculation of orbital integrals that arise in Langlands theory, using methods of mathematical logic. Kac-Moody groups and their variants were another subject addressed in the talks by Karl-Hermann Neeb (Erlangen), Manish Patnaik (Alberta), Guido Pezzini (Erlangen), and Christoff Zellner (Erlangen). Julee Kim (MIT) and Jessica Fintzen (Harvard) spoke about recent progress on the p -adic theory of supercuspidal representations and the Sato-Tate equidistribution, while Camelia Karimianpour (Ottawa) spoke on the K -types of admissible representations of a p -adic metaplectic cover. Matthew Dawson (CIMAT) spoke about invariant means and operator algebraic features of direct limit groups. The last lecture (but certainly not the least interesting!), given by Eitan Sayag (BGU), beautifully complemented the mini-courses by presenting exciting applications in both the real and the p -adic case.

Given the strong participation, we feel proud to continue the University of Ottawa's long tradition of hosting, and supporting, substantial Fields-sponsored conferences on the representation theory of Lie groups.

Monica Nevins (Ottawa) and Hadi Salmasian (Ottawa)

3rd annual Southwestern Ontario Graduate Mathematics and Statistics Conference

May 19-20, 2015 - University of Guelph

THIS SPRING the 3rd annual Southwestern Ontario Graduate Mathematics and Statistics Conference was hosted by the Department of Mathematics and Statistics at the University of Guelph, Ontario. The two day event was a huge success, having graduate students from across the province present original and exciting research being conducted throughout Southwestern Ontario. The first day consisted of a full set of talks, with 11 speakers in various fields. Following the presentations, a conference social event was held on campus at the Brass Taps, allowing participants to form collaborations and further discuss ongoing research outside their immediate research area. The second and final day showcased 6 more speakers.

The presentations covered a very wide variety of fields and topics in both mathematics and statistics, with the majority of talks falling on the mathematics side. The conference began with an introduction to variational inference followed by a talk on test functions that identify weighted composition operators and another on orthogonal separation of the Hamilton-Jacobi equation on Spaces of Constant Curvature. Subsequent presentations covered topics such as mathematical models of the spatial spread of Hepatitis C virus, rectangular Schur functions and ribbon Schur functions as well as private quantum channels and quasiorthogonal subalgebras.

Afternoon topics included ring optimization, dynamic boundary stabilization of the Schrodinger Equation and Newton-Okounkov bodies of Peterson and Bott-Samelson varieties. Following this, models of deep learning architectures for soil property prediction and cluster analysis of microbial meta-genomic data via mixtures of Dirichlet-multinomial regression models were presented.



Presentation topics of note on the second day included a model of fish-borne pollution exposure and well as a model to optimize conference success: a problem in break scheduling. An introduction to enumerating flags of a partially ordered set was also presented along with a talk on entangled symmetric multipartite qubit states and geoids in general relativity.

The conference had 40 graduate student attendees from eight universities across the province. The generous support from the Fields Institute helped provide graduate students the opportunity to present their original research and to learn about the exciting research being conducted by their peers in the region. Thank you to everyone who made this year such a success! Looking forward to seeing everyone again next year!

Marianne Wilcox (Guelph)

Inaugural Conference on Financial Econometrics and Risk Management

March 28, 2015 - Western University

The inaugural *Financial Econometrics and Risk Management* conference was held at Western University in London, Ontario, at the Ivey Spencer Leadership Centre on 28th March, 2015. Prior to the conference a half-day workshop on Market Microstructure was organized on Friday 27th March 2015, with lectures by Ingrid Lo (Bank of Canada), and George Jiang (Washington State University) and this was followed by a reception. The actual conference opened with a conference dinner for speakers, session chairs, and sponsors on the evening of March 27 and the actual conference program featured a full day of invited presentations on Saturday, 28 March 2015.

The one day program was introduced by Phelim Boyle (Wilfrid Laurier University) and consisted of seven talks, with national and international speakers drawn from leading academic and industry experts on financial econometrics and risk management. Each speaker was allocated 45 minutes, including discussion and questions. The following speakers (in order of presentations) participated:

- Tingting Lu (Highstreet Asset Management)
- Tom McCurdy (Rotman School of Management)
- Yanqin Fan (University of Washington)
- George Jiang (Washington State University)
- Bruno Feunou (Bank of Canada)
- Pujun Liu (BMO Financial Group)
- Juan Carlos Escanciano (Indiana University)

In the aftermath of the financial crisis of 2007-2009, which led to massive bank failures and bailouts and a global recession, it has become clear that the models being used to value complex financial products were not only unrealistic but also incapable of accurately measuring the risks associated with the use of these products. In order to prevent similar economic disasters in the future, we must not only develop new robust, real time financial models that accurately reflect the complexity of today's financial markets, but these models must also provide financial institutions with the ability to rapidly assess financial risk, ultimately allowing for the nimble regulatory action by bodies such as the Bank of Canada and the Office of the Superintendent of Financial Institutions Canada (OSFI).

Historically, the financial industry has relied on models that do not accurately reflect the complexity of the marketplace, and thus neither have the capacity to accurately predict the risks associated with the use of these products nor the ensuing economic crises that can occur from their use. Instead there is an urgent need to develop statistical models and frameworks, which are flexible enough to realistically describe the dynamics of financial asset returns which can take into account the correlation between multiple assets. This year's conference explored these issues in detail with talks on various topics such as investment strategies and risk management, on risk measurement, on new methods for modelling financial data

and optimally forecasting asset return variance, and on extracting information from derivatives markets.

The two events attracted approximately 60 participants from academia and from the private industry. For the workshop the majority of the participants were graduate students from Western and from surrounding universities and a high ratio of graduate student participation in this part of the event was encouraged and expected. For the conference part of the event 16 academics, 15 practitioners, and 30 students participated. The conference reached far beyond the local area with academic participants from most of the surrounding universities and with participating practitioners not only from London but also from Toronto and other centers.

The conference and half-day workshop received generous sponsorships from the Centre for Financial Innovation and Risk Management (CFIRM), the Fields Institute for Research in Mathematical Sciences, Highstreet Asset Management and the Global Risk Institute (GRI).

In all respects the conference was extremely well attended by students, faculty members and practitioners and given the generous support and amount of sponsorships received it is expected that the conference will be organized annually by Western University.

Lars Stentoft (Western)

Algorithms and Complexity in Mathematics, Epistemology and Science

May 6-8, 2015 - Western University



Robert Corless introducing Jesús María Sanz-Serna, who kicked off the conference with his talk entitled "Blind to Randomness"

Introduction: Furthering Computational Epistemology

Understanding the implications of scientific models and theories typically requires computation. In some cases the required computations are straightforward, and the theories are, in a sense, inferentially transparent. In studying classical field theories, such as general relativity, for example, exact solutions can be obtained by imposing various symmetries. Yet the known exact solutions do not by any means exhaust the content of the theory. Situations of real physical interest have to be modeled using numerical analysis, which is anything but transparent due to the complexity of the models. There is no exact solution globally describing the 2-body problem in general relativity, and a numerical simulation for (a restricted class of) 2-body systems required three decades of effort to complete. How can this two-body simulation be assessed: does it accurately reflect the "true" content of general relativity, despite approximations and simplifications in the model? How do we contrast the true content of the physical theory from artifacts of the computational methods? Similar questions arise in conjunction with computational models in a variety of fields. This project aims to articulate a comprehensive and coherent perspective on computational epistemology that elaborates on the philosophical consequences of acknowledging the central role of numerical analysis and computational complexity, in place of an assumption of inferential transparency.

Our approach focuses primarily on the implications of computational

complexity. In particular, our aim is to develop an account of theoretical knowledge in which efficient computation plays a central role. In our view, philosophers have not been sufficiently attentive to developments in numerical analysis and computational complexity. Theoretical knowledge has been too quickly regarded as consisting of an understanding of exact solutions to fundamental dynamical equations. From a computational point of view, to have theoretical knowledge worthy of the name we need to have an efficient computational algorithm to answer a range of questions, within specified constraints on time and computational resources.

Computer scientists have developed a classification of problems into different classes of computational complexity, based on the resources and operations required to solve them. Yet in many cases where one cannot find an exact solution of complex model equations, one can find an approximation to the model equations that is computationally tractable. We aim to more accurately characterize the computational tractability of scientific models by considering such approximations in characterizing computational complexity. From this point of view, the key epistemological question concerning the reliability of models and their solutions then becomes: when we don't know the exact solution to a problem, how can we determine how close a computationally tractable approximation comes to the truth?

Recent work have addressed this question using a collection of tools of analysis including, importantly, backward error analysis (BEA).

The key insight here is that the complexity of a problem does not have to be characterized in reference to the problem itself, but rather in terms of a set of problems sufficiently similar to the one given. Used properly, BEA guarantees that computational approximations



Figure 1: Jesús Sanz-Serna in action, answering questions.

are simply equivalent to generic, small, perturbations of the input data or of the mathematical model. This is an important result because it shows, contrary to the received view, that there is a close correspondence between the syntactic, the semantic, and the pragmatic components of scientific representation, inference, and computation.

One aspect of our work involves extending the range of these tools in applied mathematics. Among other things, we seek to address several classes of structured problems, starting with the numerical integration of Hamiltonian problems where some successes have already been achieved. This refinement is essential, since it is often not sufficient to have solved a nearby problem: one must solve a nearby problem of the right kind, that is, sharing structural features with the original problem (in the case of Hamiltonian problems, the structural property in question is symplecticity). Developing such applied mathematical methods illuminates the relationship between structural stability and complexity. On the philosophical side, this initiative involves considering the epistemological consequences of acknowledging the importance of computational complexity. This event has contributed significantly to the growing discussion of scientific modeling by philosophers, drawing on a richer understanding of the complexities and challenges of efficient computation.

The Conference

One of the principal aims of the conference was to foster multidisciplinary conversations between philosophers, applied mathematicians, and computer scientists, including some participants working at the intersection of the three disciplines. To that end, we had a number of participants having related interests, but approaching the key questions relating to computational epistemology from different angles. The program was structured so that researchers working in different disciplines would alternate, thus ensuring exposure to different methods and conceptual frameworks. In particular, we had an invited talk by a computational scientist and another by a philosopher on each day of the conference. Similarly, contributed talks were scheduled to alternate, in so far as possible. We are happy to report that this was successful, and there was significant engagement by the audience to many of the talks.

The conference had 54 registered participants. This number includes six invited keynote speakers, 17 speakers who have given contributed talks, as well as 31 additional registered participants. As there was no registration fee, other local scholars have also attended a few talks without official registration.

The participants to the conference had their home institutions in four

different countries (Canada, the United States, Spain, and France), making this first ACMES conference a truly international event (see figure 2). In addition, one of the objectives of the conference was to allow researchers at earlier stages of their career to present their work and interact with established applied mathematicians, philosophers of science, and computational scientists. In fact, more than 50% of the speakers at the conference were graduate students from Canada and abroad, and another 15-20% of the participants were either postdoctoral researchers or junior faculty members (see figure 3).

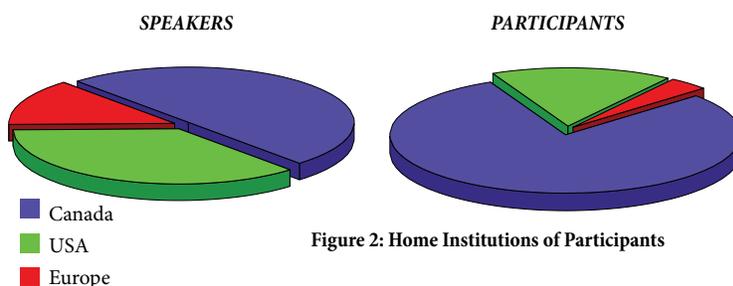


Figure 2: Home Institutions of Participants

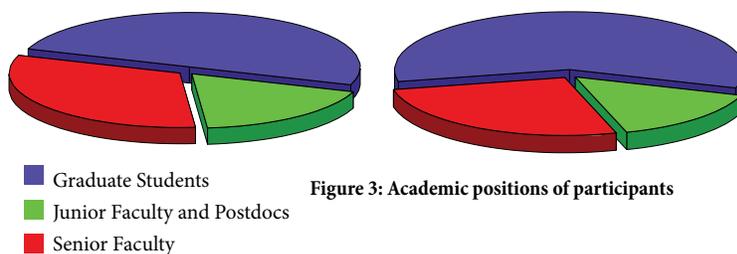


Figure 3: Academic positions of participants

This composition resulted in a constructive synergy of scholars across disciplines, countries, and stages of career development. In addition to building a multibackground network of scholars with interests in the multifaceted field of computational epistemology, our key aim was to reorient this emerging field by better delineating the fundamental problems and methods around which further research will revolve. In this respect, the organization committee judges that the conference was a great success. The questions that were identified as having the deepest relevance, which were also the most commonly raised questions in the talks and in the panel discussion, are as follows:

- In what sense or senses are computer simulations reliable sources of knowledge? How much of the justification is to be attributed to general methods of quantification of error and reliability, and how much is to be attributed to the specificity of modelling situations?
- How can one assess the reliability (or unreliability) of such simulations? In particular, what grounds the increasing use of *a posteriori* methods of analysis, and what are the implications for the direction of the field?
- Can one guarantee reliability? At what cost? In particular, what are the computational constraints that bound reliability in computational and inductive scientific practices? Consequently, what feasibility conditions should be formalized to unify logical reconstruction of modelling and theorizing practices with the unavoidable presence of uncertainty and error?
- What, if anything, do these assessment tools offer for understanding some of the old problems in epistemology? Moreover, what are the implications of new community-based proof and validation methods?

It was widely agreed that, because so much of our knowledge of the world comes from simulation nowadays, that these are compelling and stimulating questions. The (partial) answers offered in the talks and panel discussion at this conference included a deep synthesis of the relatively new and somewhat amorphously defined discipline of uncertainty quantification by Max Gunzburger, and several older strategies such as perturbation analysis, residual computation, and backward error analysis (which seems fairly well known in

numerical analysis but not so much in other disciplines, especially in philosophy). Classical ‘forward’ error analysis is of course very widely studied and remains an important tool of assessment. But the main advantage of backward error analysis, done *a posteriori*, is that it can put simulation errors on the same footing as modelling errors, which have to be considered anyway in any well-designed study of models. The importance of understanding the effects of various sorts of errors—whether they arise out of computational approximations, idealizations, or incompatible but complementary descriptions of systems for various regimes was the main theme that Mark Wilson illustrated through his philosophical and historical study of classical works in the history of applied mathematics from Euler and the Bernoullis to our days.

Several of the talks addressed randomness, its uses and abuses, and its implications in simulation. Jesús Sanz-Serna presented compelling case-analyses demonstrating the various ways in which randomness can both provide essential support and be a significant epistemological obstacles to understanding systems through simulations. The power of randomness lies in the important gain in tractability that results from the use of random algorithms to evaluate deterministic problems. The limitations he emphasized focused on the spurious appearance of rigid deterministic of causal determination when in fact subtle considerations having to do with asymptotic behaviour of stochastic processes underlie the patterns one seek to explain. In a similar vein, Robert Batterman discussed the importance of averaging methods to characterize the multi-scale behaviour of systems in the context of continuum and statistical mechanics, and their implications for our understanding of the concept of scientific explanation.

Finally, social strategies for improving reliability, such as the recent movement towards increased reproducibility in computational scientific research, were singled out. Ursula Martin discussed a social-science style analysis of the Polymath project; it was pointed out in the questions to her talk that an analogue of the polymath project in applied mathematics might be the I.P.C.C., which perhaps conveys the importance of assessing the reliability of simulation: if whole ecologies (and therefore economies) are dependent on the policies derived at least in part on simulation, the stakes are very high. Some would argue that the stakes offered in the Polymath project—the truth of a mathematical theorem—are even higher, but either way the tools for assessment of reliability become legitimate objects of study. Anouk Barberousse also reached similar conclusions, but this time from a careful study of the validation methods that underlie the multi-institution development of models in climate science.

Accomplishments and Exposure

The home of ACMES on the web is our website: <https://acmes.org/>. We will keep a record here of the speakers at the conference as well as their talk titles and abstracts. In the near future we will extend the website to include access to a repository of slides from participants

from the conference as well as links to video recordings of the talks by the invited speakers. This will ensure that the contributions of the conference have a wider impact than the event itself and will continue to be accessible in the future.

We are currently in negotiations with Springer for a publication based on the results of the conference. This will include invited and contributed papers focused on the development of a new field of computational epistemology, merging the problems and methods of philosophers and computational scientists in the examination of the reliability of computation and applied mathematical modeling.

More broadly in terms of exposure, we plan to submit a version of this report for inclusion in SIAM news. In the interest of accessibility to a wider audience, we are also considering the development of open materials that would be freely accessible online. These may include brief introductions to computational science principles and practices for philosophers, and introductions to epistemology of science approaches and perspectives for computational scientists, as well as a glossary of terminology and acronyms to assist in interdisciplinary communication.

Based on the successes of the ACMES conference, we intend to hold a future edition of ACMES in Vancouver, hosted by SFU, possibly continuing as a biannual conference. In the interest of serious multi-disciplinary collaboration, we will seek the collaboration of the Department of Philosophy, the Department of Mathematics, PIMS, IRMACS, the vice-president’s (research) office funds, as well as other institutions and university departments.

Acknowledgements

We would like to thank both the Fields Institute for the Mathematical Sciences and the Rotman Institute of Philosophy for their generous financial support of the ACMES Conference. Particular thanks to Matheus Grasselli and Esther Berzunza of the Fields Institute and Henrik Lagerlund and Rob Read of the Rotman Institute. Thanks also to the Department of Applied Mathematics for their support both financially through David Jeffrey, and administratively through Cinthia MacLean. Special thanks to Elisa Kilbourne in the Department of Philosophy for her attentive and thoughtful administrative support, which was an important contribution to the success of the conference. We would like also to thank Julia Jankowski for her administrative support and to Eunice Chan, Dan Hicks, Leili Sevyeri, Steven Thornton, and Yan Wang for their volunteer support during the conference.

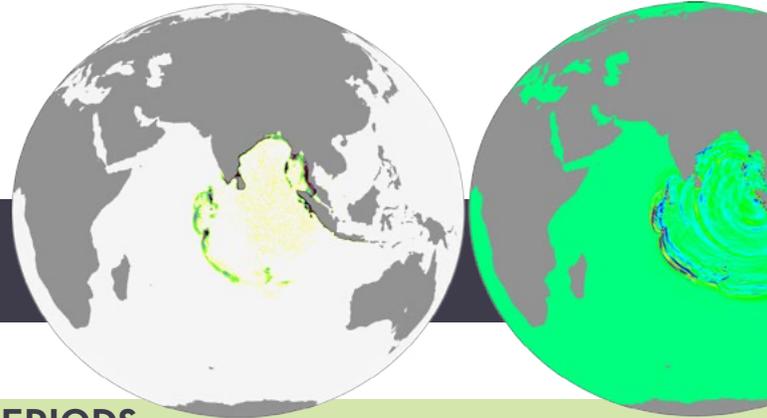
*Robert M. Corless (Western), Nicolas Fillion (Simon Fraser),
Robert Moir (Western), Chris Smeenk (Western)*



THEMATIC PROGRAM ON MULTISCALE SCIENTIFIC COMPUTING

*From Quantum Physics and Chemistry
to Material Science and Fluid Mechanics*

**JANUARY-APRIL 2016
FIELDS INSTITUTE, TORONTO**



Scientific computing is a fundamentally interdisciplinary endeavour bridging applied mathematics, application disciplines such as various physical, life and social sciences, with software and computer engineering. Benefiting from tremendous advances in (super)computer hardware, scientific computation is now acknowledged as a third method of scientific discovery, complementary to experiment and theory. While scientific computing has recently led to important advances in a number of fields, further progress is contingent on continued interaction and collaboration between different communities. Thus, the objective of this thematic program is to bring together prominent researchers (mathematicians as well as computational and application scientists) from across the globe who have contributed to the development of relevant computational methods and those who have pioneered their innovative use to address some of the fundamental problems in the following application domains:

- THEORETICAL FLUID MECHANICS
- QUANTUM PHYSICS AND CHEMISTRY
- MULTISCALE PHYSICAL PHENOMENA

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(Workshop on January 25-29, 2016)

**COMPUTATION OF QUANTUM SYSTEMS IN COLD-MATTER
PHYSICS AND CHEMISTRY**
(Workshop on February 22-26, 2016)

**MULTISCALE MODELING AND ITS APPLICATIONS:
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Workshop on Geometry of Noncommutative Manifolds

March 16–18, 2015 - Fields Institute

General description

A WORKSHOP ON THE Geometry of Noncommutative Manifolds was held at the Fields Institute on March 16–18, 2015. This workshop was supported by both the Fields Institute and the US National Science Foundation (grant DMS-1266158). The organizers were George Elliott (University of Toronto), Piotr Hajac (Institute of Mathematics, Polish Academy of Sciences), and Jonathan Rosenberg (University of Maryland). There were 30 participants, all of whom (except for the organizers and some of the invited speakers) were graduate students and postdoctoral fellows

in operator algebras, geometry, or mathematical physics. The purpose of the workshop was to focus attention on some of the main recent developments in the noncommutative Riemannian geometry of “noncommutative manifolds,” and to discuss open problems and prospects for future research. To this end, the workshop was organized around two three-lecture minicourses, by Masoud Khalkhali from the University of Western Ontario and by Henri Moscovici from Ohio State University. Khalkhali and Moscovici are two of the world’s leading experts in noncommutative geometry and both are master teachers. Their courses helped set the subject in context and provided background that graduate students and postdocs would need to get started in the field. In addition to the minicourses, there were seven additional talks, four by postdoctoral fellows (Réamonn Ó Buachalla from IMPAN, Mira Peterka from the University of Kansas, Farzad Fathizadeh from Caltech, and Branimir Ćaćić from Texas A&M University) and three by senior researchers (Piotr Hajac from IMPAN, Joakim Arnlind from Linköping University, and Ralph Kaufmann from Purdue University).

The workshop was a big success in all respects. Feedback from the participants about the minicourses was uniformly positive, all the lectures were well prepared and well received, and there was lots of informal discussion and collaboration.

Summary

What are noncommutative manifolds and why are they important? While there is no universal agreement on a definition of a noncommutative manifold, there is general agreement on the most basic examples. Aside from such manifolds which are “almost commutative,” such as Azumaya algebras over the algebra of functions on an ordinary (commutative) manifold, there are the noncommutative tori, which have attracted a huge amount of attention. In dimension 2, these specialize to the famous “irrational rotation algebras.” Other examples of noncommutative manifolds are noncommutative Riemann surfaces, the quantum group $SU_q(2)$ (which can be viewed as a noncommutative S^3), and the Podleś spheres. All of these examples have arisen naturally in the physics literature as models for various quantum phenomena, such as

compactifications of string theory and other quantum field theories in various limits, and the quantum Hall effect in condensed matter physics.

Geometry on the noncommutative 2-torus or irrational rotation algebra has now advanced to the point where there is work on analogues of many classical theorems in the classical differential geometry of surfaces. The approach that has been tried most is studying the zeta function of the “Laplacian” for a conformal deformation of a flat metric Laplacian by a conformal factor in the (noncommutative) algebra. By this method Connes and

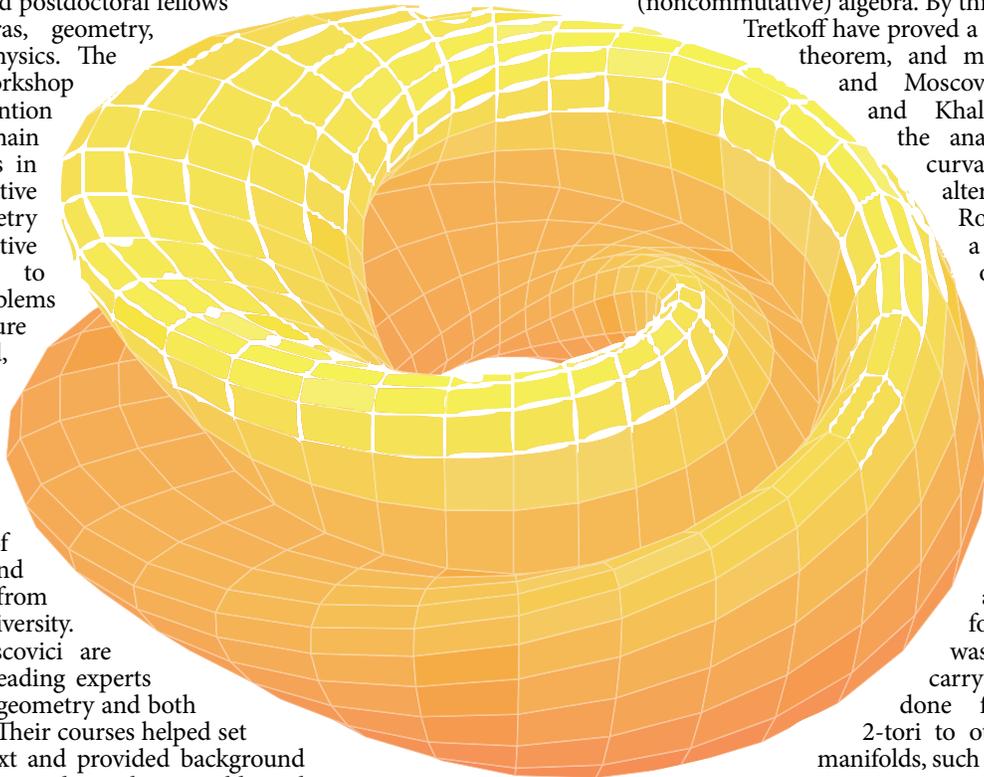
Tretkoff have proved a kind of Gauss–Bonnet theorem, and more recently, Connes and Moscovici and Fathizadeh and Khalkhali have studied the analogue of the scalar curvature function. An alternative approach of Rosenberg starts with a more general notion of Riemannian metric and gives a unique associated “Levi-Civita connection” from which various geometric invariants can be extracted. One of the purposes of this workshop was to try to reconcile these two very different approaches. Another focus of the workshop was on attempts to carry over what has been done for noncommutative 2-tori to other noncommutative manifolds, such as higher-dimensional noncommutative tori (for which there are some partial results) and noncommutative spheres and Riemann surfaces of genus > 1 .

Abstracts of the Minicourses

Henri Moscovici, *The Spectral Way in Noncommutative Geometry*

In Noncommutative Geometry the paradigm of space as a manifold formed of points labeled by numerical coordinates is replaced by one in which the coordinates are operator-valued and not required to commute, as in quantum physics. While the general topological content of such a space resides in its representation as a C^* -algebra, the extra refinement conferring on it a geometric structure is captured by Connes’ *spectral triple* template. For the global treatment of these spaces there are well-developed algebraic and analytic tools available, which have been successfully adapted and upgraded from their classical topological context. On the other hand, the local geometric concepts are much less transparent, inasmuch as they can only be accessed via spectral functionals related to the high frequency behavior of the spectrum of D coupled with the action of the algebra of coordinates, as illustrated by the local index formula. The basic notions evoked above will be sketched out in my **first lecture**.

The concept of *intrinsic curvature*, which lies at the very core of Geometry, remains quite difficult to grasp in the noncommutative framework. As the most primary form of classical curvature arises for Riemann surfaces, it was natural to look first at noncommutative 2-tori, the simplest and best understood examples



of noncommutative manifolds. The setup for the computation of the scalar curvature of the noncommutative 2-torus, equipped with a translationinvariant conformal structure, was developed in the work of Connes-Tretkoff, initiated in late 1980's. They proved the analogue of the Gauss-Bonnet formula for the conformal metric, which only required the computation of the integral of the curvature. The full calculation of the curvature was completed relatively recently, and is one of the main topics to be covered in Masoud Khalkhali's lectures. My **second lecture** will highlight the crucial role played by the Ray-Singer log-determinant functional on the space of conformal metrics. In my work with Alain Connes, we found a closed formula for this functional, and showed that its gradient coincides with the scalar curvature. Combining the two results we arrived at a more conceptual understanding of the intricate curvature formula. Furthermore, using positivity in the operator-theoretic sense, we showed that this functional, or rather its scale invariant modification, attains its extremal value only at the flat metric, an analogue of the result used by Osgood-Phillips-Sarnak to recover the classical uniformization theorem for Riemann surfaces.

In the **third lecture** I will present new results in this direction, obtained in joint work with Matthias Lesch, revealing a surprising facet of Morita equivalence, which confers the Ray-Singer functional a status similar to that of the (Connes-Rieffel) Yang-Mills functional. Our main technical tool, of interest in itself, is the extension of Connes' pseudodifferential calculus to Heisenberg bimodules.

Masoud Khalkhali, *From Spectral Geometry to Noncommutative Geometry*

One of the cornerstones of Alain Connes' noncommutative geometry is the classical spectral geometry. In these lecture series I shall start by recalling some of the key ideas of spectral geometry, starting with Weyl's asymptotic law for eigenvalues of elliptic operators on closed manifolds and its ultimate extension through Gilkey's local

formulas for heat kernel coefficients of Laplace type operators. I shall emphasize new features one encounters when dealing with noncommutative spectral manifolds. I shall then survey recent progress in understanding differential and conformal geometry of curved noncommutative tori. This is based on work of Connes-Tretkoff, Farzad Fathizadeh and myself, and Connes-Moscovici. Among other results I shall recall our computation of spectral invariants, including scalar curvature and noncommutative residues, for this class of spectral manifolds in dimensions 2 and 4.

In the final lecture, I shall present new results on rationality of spectral action for Robertson-Walker metrics (obtained jointly with Fathizadeh and Ghorbanpour), and Quillen's metric and the determinant line bundle for noncommutative two torus (joint with Fathi and Ghorbanpour). I will show how to compute the curvature of the determinant line bundle for a family of Dirac operators for noncommutative two tori. Motivated by Quillen's original construction for Riemann surfaces and using zeta regularized determinant of Laplacians, one can endow the determinant line bundle with a natural Hermitian metric. By using an analogue of Kontsevich-Vishik trace, defined on an algebra of classical logarithmic symbols, one can compute the curvature form of the determinant line bundle by computing the second variation $\delta_w \delta_w \log \det(\Delta)$.

These lectures are aimed at grad students and postdocs. In particular no background in noncommutative geometry will be assumed.

Jonathan Rosenberg (Maryland)

Big Data in Commercial and Retail Banking

May 20-21, 2015 - Fields Institute



Adam Metzler of Wilfrid Laurier University presenting his work "State-Dependent Correlations and PD-LGD Correlation - Modeling and Computation".

ON WEDNESDAY, MAY 20TH through to Thursday, May 21ST the Fields Institute was host to a very successful workshop on Big Data in Commercial and Retail Banking. The workshop attracted 44 participants made up of 24 graduate students, two postdocs, ten faculty, and eight Industry representatives. The event also had

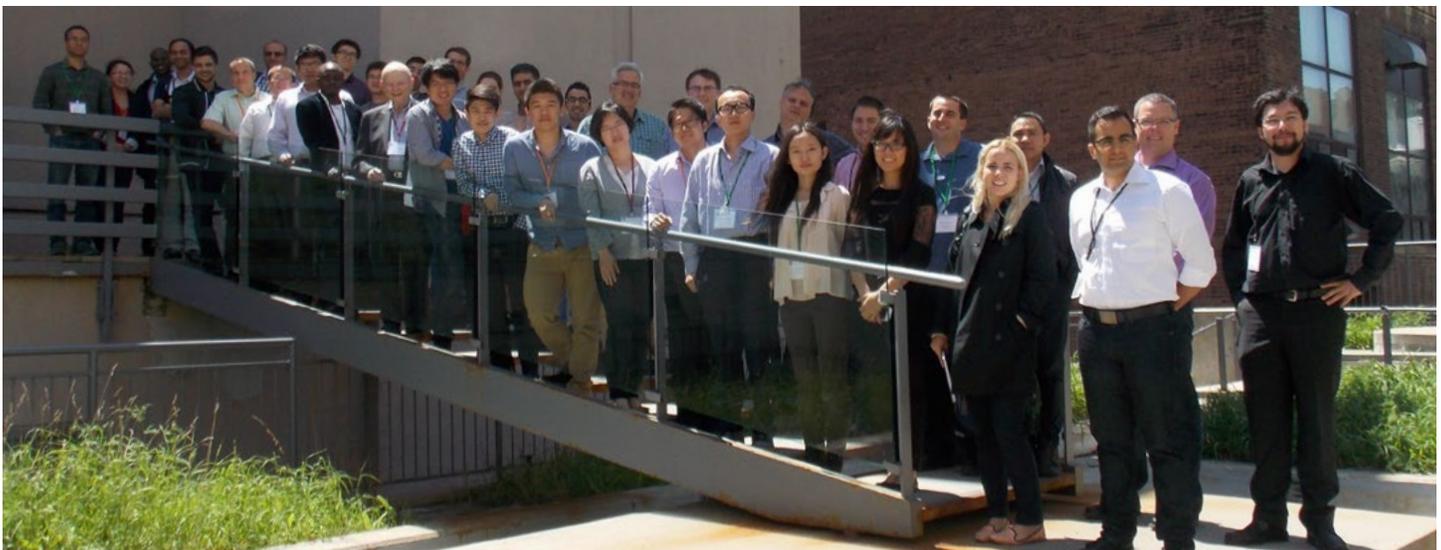
a number of other attendees that tuned in through the Institutes' FieldsLive webcast.

The workshop benefited greatly from a detailed set of lectures delivered by Professor Cristián Bravo (Universidad Talca, Chile), who is an international expert in the application of machine learning to consumer and small commercial credit. Prof. Bravo delivered eight hours of lectures which began by covering the basics of credit risk modeling, focusing attention on the key quantities of probability of default, loss given default, and exposure at default. This opening talk accomplished the near impossible dual tasks of providing an introduction to the basic business concepts making the information accessible to the credit newcomers in the room while still providing plenty of interesting insights and tidbits for the many experienced bankers and researchers present. The idea of credit scoring, with a focus on legal and operational constraints driving this process, and a very useful description of the kind of data available to make these decisions, was then introduced. During this presentation it was emphasized that existing commercially available credit scores, while useful, are not alone a sufficient data set for making good business decisions. The technical content required to operationalize these ideas, in terms of machine learning and classification, were then described, again at a level that provided something for everyone present.

On the second day of the workshop Prof. Bravo described case studies arising from the significant repertoire of industrial and practical problems he has solved over his career, bringing many useful insights to all participants. The workshop concluded with an effective lecture by Adam Metzler (Laurier) on modeling correlation between default-risky assets, with an application to modeling structured products like collateralized debt obligations.

The lead organizer of this workshop was Mark Reesor (Western) with co-organizers Matt Davison (Western) and Adam Metzler (Laurier). We thank the Fields Institute, Western Science, and Western's Department of Statistical and Actuarial Sciences for sponsoring the workshop.

Matt Davison (Western), R. Mark Reesor (Western)



Participants of the Workshop on Big Data in Commercial and Retail Banking, with workshop leader Cristian Bravo of Universidad de Talca, Chile at the far right.

Statistical Inference, Learning and Models for Big Data

January to June 2015 - Fields Institute

The thematic program on Statistical Inference, Models and Learning for Big Data focused on the study and advancement of inferential techniques for statistical learning in Big Data. The emphasis on inference was prompted by the urgent need for new statistical, computational, and mathematical research to address the ever-increasing demands of Big Data.

The program committee was appointed by the Canadian Institute of Statistical Sciences (CANSS), and allied events on the same topic took place across the country, at PIMS, CRM, and AARMS. The scientific program had two complementary strands. One strand emphasized inference and data in particular substantive areas: social policy, health policy, networks, and environmental science. The other focused on cross-cutting areas of mathematical, computational and statistical sciences, including statistical learning, visualization, optimization, and new inferential paradigms.

Several other activities at the Fields Institute were also focused on inference for Big Data, including both the 2014 and 2015 Distinguished Lecture Series in Statistics: given by Bin Yu and by Terry Speed, the Coxeter Lectures given by Michael Jordan, a workshop on Complex Spatio-Temporal Data Structures, a workshop on Big Data in Commercial and Retail Banking, a Distinguished Public Lecture by Andrew Lo, an Industrial Problem Solving Workshop, and a wildly successful and over-subscribed graduate student research day, which featured Robert Bell from ATT, Alekh Aggarwal from Google, and Kevin Murphy from Microsoft.

The training program was anchored by two graduate courses, as well as the opening workshop and bootcamp (January 9 to 23); these gave graduate students and postdoctoral fellows unique exposure to cutting edge research in a wide range of areas. There were six postdoctoral fellows in residence throughout the program; Roger Grosse was a Marsden PDF and Armin Hatefi was a Fields-Ontario PDF. PDF Einat Gil, a specialist in learning environments and technology, developed a very interesting program on Big Data for Grade 12 mathematics students which was piloted at a local high school for five weeks. All activities at the Fields Institute were streamed using *FieldsLive*, which was widely used both during the workshops and later: the archive is an invaluable research resource.

The opening workshop and bootcamp devoted one day to each of the themes of the various workshops, following two days of overview lectures. It quickly became apparent during this workshop that the breadth of application of inference for Big Data is immense, and that several common themes emerge that do indeed cut across all areas. Many speakers began their presentations with a definition or description of Big Data: key features widely identified are called the “Four V’s” -- volume, velocity, variety, and veracity. In essence we have Big Data because technology makes it possible to measure many new things in nearly continuous time. A great deal of this concerns human activity and is thus intrinsically interesting, and there is a pervasive sense that although this data is unstructured and difficult to handle, it contains valuable information. As just one of many interesting examples, David Buckeridge (McGill) discussed the problem of measuring the effect of nutrition. Three relevant databases for this problem that he is working with include a 10% sample of all products sold at convenience stores, along with their UPC codes, which can be linked to nutritional content; details from loyalty programs on household purchases; and medical records of disabilities or chronic disease such as diabetes. Shane Reese (Brigham Young) described the use of four years of twitter messages in South America, combined with an increase in the use of the TOR anonymizer, to predict insurgencies.

A detailed report of the bootcamp prepared by the postdoctoral

fellows and long term visitors identified a number of common challenges, and a number of common approaches. One set of challenges arises from the *volume*: most statistical and computational methods do not scale well, and simply fail on very large datasets. Special infrastructure, such as clusters of computers combined with parallel processing, selective sampling, and so-called divide and recombine techniques all have a role to play. Big Data may be “long”, involving a very large number of individual observations, or “wide”, involving a very large number of measurements on a relatively small number of individuals. Both raise difficulties: the former of the need to model potentially complex dependencies, and the latter a failure of traditional statistical theory and methods. Challenges arising from the *variety* of Big Data include the need to develop new methods for the analysis of text, images, video and networks. Spatial and time-series data may be highly heterogeneous. For example spatial data on human populations are in some databases available at the level of postal code, and in others at the level of census tract. Data on historical records of climate are constructed by combining information from ice cores, tree rings, pollen, and surface temperature records. Challenges associated with *veracity* include both data quality and data privacy. There are some high-profile failures of large datasets, such as [Google Flu Trends](#) to provide correct inferences, due to hidden or unacknowledged biases. This is partly related to the problem that a great deal of Big Data is observational, or even administrative, so was not collected with the purpose of addressing particular scientific concerns. It is also partly related to the emphasis in some fields on a ‘black-box’ approach to prediction, without developing a modeling strategy that describes important features of the data. These black-box approaches are very useful for dealing with the *velocity* of Big Data; for example for recommender systems with online viewing or purchasing, but must be used with care if the veracity of the data is in doubt. There have also been some high profile failures of privacy, by linking various databases. Privacy has long been a concern of government statistical agencies and health records agencies, and is now engaging the computer science and cryptography communities.

Common solutions to many of these problems include building more complex models, assuming underlying sparsity, developing non-convex optimization techniques, developing new visualization tools and developing new asymptotic theories. All of these approaches were developed and extended in the various workshops.

Highlights from Optimization workshop:

The workshop on Optimization and Matrix Methods for Big Data (Feb 9 – 11) highlighted several recent advances in new algorithms for data mining that come with strong statistical guarantees. One theme of the workshop was the recent emergence of non-convex optimization methods for Big Data. Conventionally, non-convex optimization comes with relatively weak guarantees, but P. Loh and A. Anandkumar showed two important application areas in which non-convex optimization is guaranteed to be efficient and also to recover the hidden structure in the data. Another emerging theme addressed by Q. Berthet, B. Recht and other speakers is the limit of our ability to carry out inference: although a signal may be present in the data and detectable in the information-theoretic sense, its detection may be beyond the range of tractable computation. Finally, many speakers discussed the crucial challenge of adapting existing algorithms for the case of really massive data sets. The workshop boasted participants from universities in Canada, the U.S., and Europe as well as government labs (Sandia) and industry (Microsoft and IBM).

Highlights from Visualization workshop:

The workshop on Visualization (Feb 23 – 27) brought together experts from the research communities in information visualization, in scientific visualization, and in statistical graphics. Visualization is an interesting problem for Big Data because our current visual devices can display only a small portion of the data, so the question becomes what to present, how to summarize the data, and how to develop interactive displays that permit multi-scale views of a given database. Sheelagh Carpendale, founder and director of the Innovations in Visualization Laboratory of the University of Calgary described new types of interactive graphical display, some modelled on fractal scales in mathematics and in nature. The Papilio display develops a visual display appropriate for a data structure that includes a partial ordering. Katy Borner, Director of the Cyberinfrastructure for Network Science Center at Indiana University emphasized the role of visualization in engaging the viewer and creating new conversations about science, technology and innovation that permits interested non-specialists to engage in conversations around Big Data.

Hadley Wickham (RStudio) and Di Cook (Iowa State) have, with their students and collaborators, revolutionized the graphical presentation of statistical data, and the organization of data bases for both plots and for statistical inference, with an original and important suite of R packages, including *dplyr*, *ggplot2*, *ggvis*, and *tidyr*. A series of presentations of ongoing work by graduate students in Di Cook's laboratory was a highlight of the workshop. One interesting aspect of visualization, especially interactive visualization, is the difficulty of capturing the dynamic nature of the process in a static publication. Thus developing tools to record the steps of an investigation play an important role, and this resonates well with the increasing emphasis on reproducible research in scientific work. RStudio, in particular, makes the careful recording of work flow much easier than it has been in the past, and this has important uses far outside the application to visualization.

Highlights from Health Policy workshop:

The workshop on Big Data in Health Policy (Mar 23 – 27) focused on causal inference techniques for healthcare data. Causality is hard to establish in large databases, where a massive number of variable associations can often be explored. Moreover, traditional

approaches for causal inference, such as regression adjustment and stratification, have limitations in Big Data environments. Speaker presentations focused on graphical methods, research designs, and inferential techniques that can be used to explore causal relationships in population health and healthcare use. Two mini-workshops, on the topics of propensity score and marginal structural models, were led by Drs. Peter Austin (University of Toronto) and Erica Moodie (McGill University), respectively. A diverse range of Big Data applications were explored in such areas as medical device surveillance, drug safety and effectiveness, and micro-simulation modeling of population health. Speakers were from several provinces in Canada, the US, and Europe.

Highlights from Social Policy workshop:

The workshop on Big Data in Social Policy (Apr 13 – 16) explored the interface between social and decision information theory and policy. Workshop days were devoted to official statistics, network models and agent-based modeling, living analytics and privacy, and urban analytics. The spiraling cost of carefully collected survey and census data is pushing governments to increase their emphasis on administrative data collected for other purposes, including taxation, for example. Robert Groves, former director of the US Census, emphasized the many policy challenges facing government statistical agencies; this is sometimes called the 'all data revolution'. While this all data revolution provides new sources of data to inform policy, it also requires changes in policy, and these changes require strategies based on statistical reasoning. Eric Miller (Toronto) described the use of agent-based microsimulation models for modelling, for example, travel demand and to experiment with the effect of alternative transportation policy decisions on travel demand and travel patterns. Archan Misra (Singapore Management University) described the "Living Analytics" project there, which is using mobile technology on campus to understand human behaviour, including social networks, queuing episodes, and indoor movement.

Nancy Reid (Toronto)

Big Data Industrial Problem Solving Workshop

May 25-29, 2015 - Fields Institute

The Big Data Industrial Problem Solving Workshop (BDIPSW) was a part of the Thematic Program on Big Data (January – June 2015) organized by Nancy Ried (Toronto). The organizers of the BGIPSW were Paul McNicholas (McMaster), Huaxiong Huang and Tyler Wilson. The workshop followed the format of the IPSW (also known as Study Groups with Industry), originated by Oxford University in 1968. The BDIPSW was a five day workshop with four problems presented on Monday (May 25), followed by three-and-half days of brainstorming and problem solving, and concluding with a final presentation on Friday (May 29).

There were approximately 50 participants at the workshop, including problem presenters from DBRS, TMX Group, GSK, and the Canadian Sports Institute, as well as academic researchers and students from Canada and beyond. The DBRS is in the business of providing credit ratings based on available financial and accounting data, and the problems posed were in this direction. TMX Group operates the stock exchanges in Canada as well as derivative and other financial markets; the problems concerned identifying causal factors creating price volatility differences between highly traded stocks and those traded less frequently. The GSK problem concerned identification of rare adverse drug events and low drug

exposure prevalence with small sample sizes. The Canadian Sports Institute brought to the workshop a large amount of training and match data related to the Canadian Women's Rugby Sevens team, which had just won the World Rugby Women's Sevens Series. They were interested in, *inter alia*, using GPS data to study player movement and working towards in-game monitoring to assist coaches with strategic decision making.

During the week, participants split up into groups to work on these problems. They faced many technical challenges that are common in solving real-world problems. For example, in the TMX room, the group discovered that data pre-processing can take a significant amount of time. In the rugby room, the group gained some appreciation for the many challenges in dealing with very large quantities of data. Despite the technical issues, progress was made on most of the problems and follow-up collaborations are underway for the rugby and TMX problems that were posed.

Huaxiong Huang (York and Fields)

Short-Thematic Program on Delay Differential Equations

May 4-29, 2015 - Fields Institute

The short thematic program was coordinated around four interconnected sub-themes, and there was substantial heterogeneity in coordination. Each of the first three sub-thematic workshops typically consisted of a “short introductory lecture”, an “opening talk” covering the main issues of the sub-theme in a format suitable for general participants, a number of “invited talks” to cover the subject in depth, “problem sessions and panel discussions” with focusing on future directions, and some “public lectures” focusing on applications. The final week was deliberately under-organized to encourage self-organized activities, so this week had only a few talks scattered through the week and helped set the tenor of the discussions.

Professor John Mallet-Paret (Brown) gave the Distinguished Lecture Series associated with the thematic program. His first lecture introduced the general area of delay differential equation and highlighted two major issues, namely, existence of periodic solutions and the Morse-Smale structure of global attractors. His second lecture, along with the Opening Talk by Roger Nussbaum (Rutgers) for the workshop *Differential Equations with Variable Delay*, addressed ongoing research about coexistence of analyticity and non-analyticity of solutions, a recently observed phenomenon which seems due to variable delay. In the Introductory Lecture, Tibor Krisztin (Szeged) explained the basic theory which is specific for equations with variable, and notably, state-dependent delay, which are not covered by the more familiar results for functional differential equations with constant delay, distributed or not. Further lectures in this workshop were devoted to quite different topics including regularity of solutions, complicated dynamics due to variable delay, the Wheeler-Feynman system for the n-body problem of electrodynamics, the nonlocal Vlasov-Poisson system for rarefied plasma, applications in population models, numerical methods tailored for equations with variable delay, and a unifying approach to bounded and unbounded delay which results in a semiflow on a submanifold of a Frechet space. The problem sessions focused on these open questions for equations of neutral type, on systems related to population biology, and once more on the Wheeler-Feynman system.

The second workshop *Delay Differential Equations in Physical Sciences and Engineering* included the opening talk of Ingo Fischer (Palma) that addressed the “nuisance, challenges and opportunities” of delays in physical systems, and the public Lecture “Delay models for dynamic contact problems: machine tool vibrations” by Gabor Stepan (Budapest). This workshop was in the tradition of previous conferences on delayed complex systems and applications of delay equations in physics and engineering (Berlin 2006, Dresden 2009, Palma de Mallorca 2012). It has now become a mature field, which continues to thrive and grow. The panel discussion, with Ingo Fischer, Daniel J. Gauthier, Sabine Klapp, Yuliya Kyrchko, Eckehard Schöll and Gabor Stepan as the panelists, was directed to address a few pre-identified questions. The first was “which extensions of the theory are needed?”. The panelists and participants concluded that there is a particular need of further mathematical developments in state-dependent delay. This includes distributed delay, time-varying delay (including periodic as well as stochastic delays), and adaptive delay. Associated with state-dependent delay is the study of hybrid switching systems. The next question addressed was “to which new fields should delay differential equations be applied”? It was agreed that a general aim is to deliberately design delay systems for useful applications, for suppression or stabilization of unstable states, or for generation of new states. In the last couple of years, new fields of applications have emerged, namely soft condensed matter and biological systems, and quantum systems, where time-delayed feedback control has only very recently been applied, and further investigations appear very promising. Likewise, the climate is a complex system where time delays

should be studied systematically, as are networks with complex topology (e.g., hierarchical, multipartite, multi-layer, multiplex, networks of networks), and multi-scale problems. Another timely field of applications for the future is information processing, e.g., reservoir computing. This panel discussion suggested (and echoed by all other thematic program participants) that there is a lack of knowledge in our university teaching. The field should be introduced in undergraduate and graduate teaching, i.e., special courses on “delay differential equations” should be offered.

The third workshop *Structured Delay Systems* focused on delay equations as they arise from population models or from models for the spread of an infectious disease in a host population. There were 21 lectures of 30 minutes, an introductory lecture “Non-local delayed systems” by Xingfu Zou (Western), and the John Mallet-Paret’s third Distinguished Lecture on “Tensor products, positive operators and delay-differential equations”. 12 Elevator Pitches and 5 Posters were also presented. Apart from discussions after (or during) lectures, there was much exchange going on in smaller groups. The following three topics were discussed in somewhat larger groups: how to build tools for the numerical bifurcation analysis of general structured population models (nicknamed the ‘Daphnia’ topic); epidemic models; and the connection between the partial differential equation formulation and the delay equation formulation. The workshop served as a positive feedback loop to strengthen the abilities of the participants to cope with the “three hats problem” viz., that work in this area involves model formulation, model analysis, and model application (first identification, i.e., parameterization of the model on the basis of data, and next interpretation of the results in the motivating context). From a more mathematical point of view, the workshop helped to bridge the gap between the dynamical systems perspective and the numerical (bifurcation) analysis perspective for delay equations.

The final workshop *Delay Differential Equations in Life Sciences and Medicine* was kick-started by a 3 minute speed talk (without slides) by everyone attending, and this seemed to help break the ice and introduce people to one another. In a group discussion participants highlighted a number of possible working groups that people were interested in. These were: Networks & delays; Model/data fit; Numerical analysis; Pharmacokinetics and pharmacodynamics; Eigenvalue equations; Delay equations/partial differential equations connections; Distribution of delays; Transient dynamics and platelets. As might be expected some of the groups got off the ground, and some didn’t. But, all in all, we thought that it worked pretty well. Such is the nature of self-organization. Michael Mackey (the organizer of this week) wrote “I first explained in detail a physiological model of red blood cell production to Morgan and Xiaotian, and how I have chosen it. Then they show me two different kinds of general pharmacodynamic models they worked on. We had a discussion about it and put all things together to make a pharmacokinetic/pharmacodynamics model of hemoglobin regulation through synthetic Epo treatment. They also explained me the basics of how to construct a pharmacokinetic model and the different step for it.” A participant wrote to the organizer “Following group discussions, I identified a number of attendees who have tackled problems similar to mine. You’ve struggled with something very close, i.e. dealing with stochasticity in delay equations – some discussion and an interesting problem have come out of that. I shared with Gail Wolkowicz and her student a small numerical tool I had put together to visualize the evolution of eigenvalues as the p parameter of the Gamma distribution is increased. Interesting discussions with Jan Sieber and Fatihcan Atay brought attention to questions that would be worth looking into. And a discussion with John Milton generated interest in my SDDE integrator”.

The platelet group, with members from California, Lyon, Montreal and York, had an advantage in that they came to the thematic program somewhat organized to start with. There were four major problems this group solved during the thematic program:

- The issue of the exact model formulation generated a lot of discussions between the two major architects, Gabriel and Lois. As a consequence of these discussions it was decided to stick with the somewhat simpler model in which all megakaryocyte compartments were lumped together regardless of ploidy class--a simplification to be sure, but one that was dictated by the lack of any quantitative physiological information that would have allowed the group to actually deal with parameter estimation if ploidy distribution was accounted for.
- The linear analysis of the model was cleaned up when an algebraic error was discovered. The correction of this led to the bifurcation analysis accurately predicting the location of a Hopf bifurcation (seen numerically) and the predicted Hopf period matching the numerically observed one.
- The group managed to redo the parameter estimation for the model and have now successfully managed to find or estimate all but 3 of the parameters from existing data on normal humans.
- One of the issues that has plagued the group over the past year was how to actually use the model to numerically try to predict the data the group has available from patients with cyclical thrombocytopenia. The group is hopeful that the method shown to them by Jiguo Cao will be a better alternative than

what the group has tried in the past, and Jingu has joined forces with the group to do this.

Participants of the thematic program certainly “have taken advantage as much as possible of the great opportunity to have this program, with some of the most accomplished scientists in the area, along with bright students”. As hoped, the discussions provided a rather comprehensive picture of the current status including most recent progress and a global picture of the frontiers and future directions in nonlinear dynamics of delay differential equations.

Report from the program committee: Sue Ann Campbell (Waterloo); Odo Diekmann (Utrecht), Stephen Gourley (Surrey), Yuliya Kyrychko (Sussex), Michael Mackey (McGill), Eckehard Schöll (TU Berlin), Hans-Otto Walther (Giessen), Glenn Webb (Vanderbilt), and Jianhong Wu (York).

Delay Equations in Electrodynamics

Article

The study of time evolution equations that involve terms which are not only dependent on the same time instant but on delayed times are found in many areas of applied mathematics ranging from physics over biology to computer science and engineering. Often these delayed terms stem from modeling a process that microscopically may not even involve delays. Examples are effective descriptions of pressure waves in many-particle systems, the dispersion of ions in a neuron, the average reaction time lag of users interacting in a computer network, or the coupling of vibration modes of a cutting tool to bumps in the surface of a spinning workpiece that were caused in previous revolutions.

As compared to standard differential equations, already very simple of such so-called delay equations bring along quite a rich mathematical structure. When introducing a delay into a system of ordinary differential equations, the uniqueness, regularity and stability properties change immediately. For instance, a well-defined initial value problem to gain solutions on the future half-space usually requires prescribing at least parts of the history of the solution instead of only Cauchy data, and if the evolution into one time direction smoothens the regularity of the initial history, the regularity often becomes worse evolving the other way round.

In contrast to systems in which delays occur because of our effective descriptions, there are also systems that involve intrinsic delays. The prime example is the electromagnetic interaction due to the finiteness of the speed of light – a physical property that, from today’s perspective maybe surprisingly, was already argued for by Empedocles around 460 BC, who reflected on the nature of light. The list of natural philosophers engaging into the controversy about it seems to include almost every well-known name from Aristotle, Euclid, Kepler, Descartes, Fermat to Galilei until, in 1676, Rømer from the Paris Observatory reported on the following observation. The times when Jupiter’s innermost moon Io emerges or plunges into Jupiter’s shadow deviate from the predicted times that were inferred from several months’ earlier observation and the known period of Io. Rømer argued that the measured time difference is due to the difference in the distance that light coming from Io had to travel to Earth given the two different positions of our planet on its orbit around the sun. Huygens and Newton improved Rømer’s estimate, and in Opticks, Newton reported that it would take light roughly 7-8min to travel from Sun to Earth which is pretty close to today’s accepted average value of about 8min and 19s.

Direct action vs. propagation

Clearly, to describe electrodynamic interaction, the idea of an action-at-a-spatial-distance in the sense of Newtonian mechanics had to be given up. In 1845 Gauß wrote to Weber desiring explicitly for an “action, not instantaneous, but propagated in time in a similar manner to that of light”. As in special relativity, whose development was triggered with the one of electrodynamics, the relevant notion of distance is given by the indefinite Minkowski metric on 3+1 dimensional space-time. For a space-time four-vector x^μ , $\mu = 0, 1, 2, 3$, where x^0 denotes the time coordinate and $\vec{x} = (x^1, x^2, x^3)$ denotes the triple of space coordinates, the square of its distance to the origin is defined by $x^2 = x_\mu x^\mu = (x^0)^2 - |\vec{x}|^2$; using Einstein’s notation summing over same Greek indices w.r.t. the metric tensor $g = \text{diag}(1, -1, -1, -1)$, i.e., $x_\mu b^\mu = \sum_{\mu=0}^3 g_{\mu\nu} a^\nu b^\mu$. This notion of distance assigns all space-time points x^μ and y^μ that can reach each other with the speed of light, a zero distance; here, in natural units the speed of light equals one. Based on the principle that action may only occur between space-time points having Minkowski distance zero, Tetradé [14], Fokker [11], and Schwarzschild [13] formulated the theory of electromagnetic interaction between N charges on world-lines

$z_i^\mu : \mathbb{R} \rightarrow \mathbb{R}^4$, $\tau \mapsto z_i^\mu(\tau)$, $i = 1, 2, \dots, N$, in terms of a variational problem involving the so-called action

$$S[z_{i=1, \dots, n}] = - \sum_{i=1}^N \int m_i \sqrt{dz_{i\mu}(\tau) dz_i^\mu(\tau)} - \sum_{\substack{i=1 \\ k \neq i}}^N \frac{e_i e_k}{8\pi} \int dz_{i\mu} \int dz_{k\nu} \delta((z_i(\tau) - z_k(\sigma))^2).$$

The informal integral $\int dz_i^\mu$ is to be interpreted as the line integral $\int d\tau \dot{z}_{i\mu}(\tau)$, where the dot denotes a derivative w.r.t. the parametrization τ . The symbol δ denotes the one-dimensional Dirac delta distribution, and m_i and e_i the mass of charge of the i -th particle, respectively. The first summand above measures the arc length of the i -th world-line using the Minkowski metric, and the second summand gives rise to an interaction between two world lines whenever the Minkowski distance between z_i and z_k is zero. The extrema of the action S , i.e., z_i such that $S[z_i + \epsilon \delta z_i]_{\epsilon=0} = 0$, are attained for world-lines fulfilling for $i = 1, 2, \dots, N$:

$$m_i \ddot{z}_i^\mu(\tau) = \frac{e_i}{2} \sum_{\substack{k=1 \\ k \neq i}}^N \sum_{\pm} F_{k\pm}^{\mu\nu}(z_i(\tau)) \dot{z}_{i\nu}(\tau) \quad (1)$$

with $F^{\mu\nu}(x) = \partial/\partial x_\mu A^\nu(x) - \partial/\partial x_\nu A^\mu(x)$ and

$$A_{k\pm}^\mu(x) = e_k \frac{z_{k\pm}^\mu}{(x - z_{k\pm})_\nu \dot{z}_{k\pm}^\nu}, \quad z_{k\pm}^\mu = z_k^\mu(\tau_{k\pm}),$$

$$x^0 - z_k^0(\tau_{k\pm}) = \pm |\vec{x} - \vec{z}_k(\tau_{k\pm})|. \quad (2)$$

Equation (1) is the special relativistic form of Newton’s force law, in electrodynamics referred to as Lorentz equation. The force tensors $F_{k+}^{\mu\nu}$, $F_{k-}^{\mu\nu}$ equal the so-called advanced and retarded electrodynamic Liénard-Wiechert field tensors, respectively, which are the electromagnetic fields generated by the k -th charge. They are given in terms of the corresponding potentials $A_{k\pm}^\mu$, which are functionals of the world line $\tau \mapsto z_k(\tau)$ since the parameters $\tau_{k\pm}$ are given only implicitly as solutions to (2). The latter equation reflects the presence of the delta distribution in S and has the nice geometrical interpretation mentioned above and depicted in Figure 1.

When evaluating $F_{k\pm}^{\mu\nu}(x)$ at $x^\mu = z_i^\mu$ as in the Lorentz force (1), the respective $\tau_{k\pm}$ identify the space-time points $z_{k\pm}^\mu$ which can be reached from $x^\mu = z_i^\mu(\tau)$ with speed of light. The existence of both, τ_{k+} and τ_{k-} , is ensured as long as the world lines have velocities smaller than the speed of light. Their values are however not bounded a priori, and since computing $F_{k\pm}^{\mu\nu}(x)$ involves taking another derivative of $A_{k\pm}^\mu(x)$ w.r.t. to x^μ , and $\tau_{k\pm}$ depends on x^μ , the right-hand side of the Lorentz equations (1) involves advanced and retarded four-vectors $z_{k\pm}^\mu$, four-velocities $\dot{z}_{k\pm}^\mu$, and four-accelerations $\ddot{z}_{k\pm}^\mu$. Hence, (1) is a neutral equation of mixed-type with state-dependent and unbounded delay for which very little is known mathematically. In very special situations some solutions are known, but in general, existence and uniqueness still remain open after almost a century. Yet it is even unclear of what type the initial data should be.

At first sight it may seem remarkable that this action-at-a-space-time-distance theory (1) described by the action S involving only simple geometric quantities describes classical electrodynamics. On the other hand the action describing Newtonian gravitation has an intriguingly similar structure. This similarity becomes apparent when noting that here, S introduced the interaction through $\delta(x^2)$, the Green’s function of d’Alembert operator $\partial_\mu \partial^\mu = \partial^2 / \partial x_0^2 - \Delta$, whereas the corresponding action for Newtonian

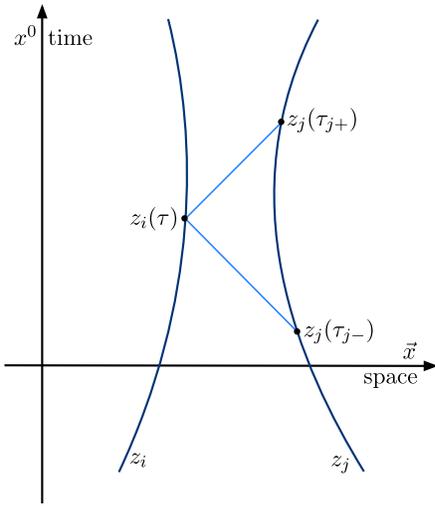


Figure 1: This figure shows two world-lines z_i and z_j . The space-time points $z_j^\mu(\tau_{j\pm})$ are found by intersecting the forward and backward light cones (their edges depicted in light blue) of $z_i^\mu(\tau)$ with the world line z_j . To compute the acceleration $\ddot{z}_i^\mu(\tau)$ according to (1), it requires knowledge of the world-line data $z_j^\mu(\tau_{j\pm})$, $\dot{z}_j^\mu(\tau_{j\pm})$, and $\ddot{z}_j^\mu(\tau_{j\pm})$.

gravitation involves the Green's function of the Laplace operator Δ . In the same sense as the Poisson equation is the simplest differential equation in Euclidean space respecting the Galilei symmetry, the wave equation is the simplest one respecting the Lorentz symmetry in Minkowski space-time. Thus, $A_{k\pm}^\mu$ plays the same role as the gravitation potential in Newton's theory, and one finds $\square A_{k\pm}^\mu(x) = -4\pi j_k^\mu(x)$ and the so-called gauge restriction $\partial/\partial x^\mu A^\mu(x) = 0$ for the charge-density-current $j_k^\mu(x) = e_k \int dz_k \delta^4(x - z_k)$. Thanks to this intimate connection to potential theory it is possible to recast the equation of motion (1) into a system of equations

$$m_i \ddot{z}_i^\mu(\tau) = \frac{e_i}{2} \sum_{k=1, k \neq i}^N F_k^{\mu\nu}(z_i(\tau)) \dot{z}_{i\nu}(\tau), \quad (3)$$

$$\square A_k^\mu(x) = -4\pi j_k^\mu(x), \quad \partial/\partial x^\mu A^\mu(x) = 0 \quad (4)$$

that have to be solved simultaneously [5]; the equations in (4) are the so-called Maxwell equations. One observes that the world-lines $z_i^\mu(\tau)$ which solve (1) provide a solution to (3)-(4) simply by setting $A_k^\mu = \frac{1}{2}(A_{k\pm}^\mu + A_{k\mp}^\mu)$, however, the system (3)-(4) admits more solutions than that. In fact, it renders a well-defined initial value problem for initially prescribed positions and momenta of the charges and sufficiently regular initial electric and magnetic fields $\vec{E}_{k,0} = -\nabla A_k^0 - \partial/\partial x^0 \vec{A}_k$ and $\vec{B}_{k,0} = \nabla \wedge \vec{A}_k$, respectively. The advantage of (3)-(4) over (1) is that the new system of equations does not involve delayed terms. The disadvantage, however, are the infinitely many additional field degrees of freedom.

Leaving the difficult ontological consequences in prescribing initial fields, being entities that extend infinitely in space, inaccessible to the experimenter, aside, the mathematical conclusion exemplifies another peculiarity of delay equations: If the delayed terms obey a certain dynamical law (here, the wave equation), the original delay equation may be recast in terms of a set of equations on a bigger phase space involving the original degrees of freedom (here, the world lines $z_k^\mu(\tau)$) and in addition new ones encoding the propagation of the delay (here, the fields $A_k^\mu(x)$).

Historically, it seems that the formulation of electrodynamics in terms of world-lines and fields as in (3)-(4), e.g., in Maxwell's monolithic work in 1861, was even recorded before the formulation in terms of world-lines only as in (1) appeared in the classical literature, e.g., in the work [13] of Schwarzschild in 1903. It seems unclear whether this was due to the mathematical difficulties that come along with delay equations or whether it was

due to the competition between the two ontological pictures of either action-at-a-space-time-distance or propagation of interaction in time. The latter picture dominated until the work [15] of Wheeler and Feynman in 1949 where they address the two fundamental problems in electrodynamics (see also [7]): 1) How can the irreversible nature of radiation be deduced from the fundamental equations of motion that are time-symmetric? 2) As accelerating charges radiate, they ought to experience an instantaneous force of damping in order to suffer a decrease in kinetic energy that equals the amount of energy in the radiated fields. What is the mechanism that provides this behavior? Especially point 2) has plagued electrodynamics since the beginning. In order to introduce a back reaction from the just radiated fields to the respective radiating charge, Abraham [1] regarded a force law that extends the summation on the right-hand side of (3) to also include $k = i$, i.e., the field $F_i^{\mu\nu}$ generated by charge $z_i^\mu(\tau)$ was allowed to react back on it. While informally, this makes sense regarding energy conservation, formally the resulting equations of motion become ill-defined. This can already be seen from explicit inhomogeneous solutions $A_{i\pm}^\mu(x)$ which are singular at $x = z_i(\tau)$, hence $F_i^{\mu\nu}(z_i(\tau))$ is, and the right-hand side of the Lorentz force equations is ill-defined. This is the origin of the infamous ultraviolet divergence that was later also inherited by quantum electrodynamics. To circumvent the divergence, Abraham replaced the point-like charges by smoothly extended charges of radius $\epsilon > 0$. This mends the singular behavior, however, breaks Lorentz invariance and introduces an arbitrary parameter ϵ into the theory. At the end of any computation it is therefore desirable to again remove ϵ by a limit procedure $\epsilon \rightarrow 0$. For this purpose, physicists regard solutions in terms of a series expansion in ϵ . As can be expected, when ϵ tends to zero, the original divergence shows up again. Nevertheless, as shown by Dirac in 1938 [4], the term carrying the divergent behavior gives only rise to a positive correction of the bare inertial mass m_i of the charge i , a so-called electrodynamic mass. Sending the bare inertial mass m_i to $-\infty$ at a rate in ϵ that keeps the total inertial mass fixed to the experimental value, one can informally keep computations finite. As strange as this idea of mass renormalization may seem mathematically, it became the role model for the renormalization of all masses and coupling constants in the standard model of quantum field theory.

Contrary to the works of Abraham and Dirac, Wheeler and Feynman reconsidered (1) which does not include such the ill-defined self-interaction term $k = i$. Assuming a large number of charges and special initial conditions in which one charge experiences an initial force while the others are in a thermal equilibrium, ensuring that the retarded fields average out, they argued that radiation reaction enters into the dynamics naturally due to the accumulated interaction of the advanced fields $\sum_{k=1, k \neq i}^N F_k^{\mu\nu}$, and does not have to be artificially added by hand. They showed that the qualitative behavior of corresponding solutions to (1) can be described by the effective equation

$$m_i \ddot{z}_i^\mu(\tau) = \frac{2}{3} e^2 (\ddot{z}_i^\mu \dot{z}_i^\nu - \ddot{z}_i^\nu \dot{z}_i^\mu) \dot{z}_{i\nu}(\tau) + \sum_{k=1, k \neq i}^N F_{k-}^{\mu\nu}(z_i(\tau)) \dot{z}_{i\nu}(\tau), \quad (5)$$

which is the same formula Abraham and Dirac found earlier employing the informal procedure of mass renormalization. The first term describes the radiation reaction and the second the interaction with the retarded fields of the respective other charges. Wheeler and Feynman's derivation addresses both questions 1) and 2) at the same time. First, noting that the effective equation is purely retarded, their derivation explains how the time-symmetry of the fundamental equations of motions can be broken by the choice of initial conditions to describe irreversible behavior in the spirit of statistical mechanics and without the need of an electrodynamic arrow of time. Second, no singularities are encountered in their derivation of the radiation reaction terms, which is due to the fact that in (1) the ill-defined self-interaction term is not present in the first place. This fact makes their approach particular interesting for a mathematical rigorous derivation of (5) to be carried out in the future once the solution theory of (1) is better understood.

Nowadays, equation (1) is usually called Fokker-Schwarzschild-Tetrode

(FST) or Wheeler-Feynman or equation while (5) is referred to as Lorentz-Abraham-Dirac equation. Since the radiation damping term depends on the accelerations of the i -th particle and the square of its charge, which for instance in the case of the electron is very small, it can usually be omitted from (5) in regimes of small accelerations. The resulting equations are then referred to as Synge equations. All three discussed equations are neutral in 3+1 dimensional space-time and involve unbounded delays which, as indicated earlier, pose many mathematical difficulties. Let us look at some of them more closely.

A toy model

For this purpose we regard a toy model for two charges that approximates (1) for small velocities and accelerations. Recasting (1) using coordinates $z_i^\mu = (t, \vec{q}_{i,t})$ for $i = 1, 2$ and dropping all terms involving velocities and accelerations yield the following system of equations

$$\dot{\vec{q}}_{i,t} = \frac{\vec{p}_{i,t}}{\sqrt{|\vec{p}_{i,t}|^2 + m_i^2}}, \quad i, j = 1, 2, i \neq j, \quad (6)$$

$$\dot{\vec{p}}_{i,t} = e_i e_j \sum_{\pm} \frac{\vec{q}_{j\pm} - \vec{q}_{i,t}}{|\vec{q}_{j\pm} - \vec{q}_{i,t}|^3}, \quad (7)$$

$$\vec{q}_{j\pm} = \vec{q}_{j,t_{j\pm}}, \quad t_{j\pm} = t \pm |\vec{q}_{i,t} - \vec{q}_{j,t_{j\pm}}|. \quad (8)$$

Equation (6) defines the relativistic velocity through the charge momentum $\vec{p}_{i,t}$, equation (7) is the simplified version of (1), and (8) defines the delay analogous to (2). These equations of motion are very similar to Newtonian gravitation, except for the fact that the choice of e_i allows to treat both, the attractive and repulsive case, and the difference $\vec{q}_{j\pm} - \vec{q}_{i,t}$ entering the inverse square law refers to the advanced and delayed positions, \vec{q}_{j+} and \vec{q}_{j-} .

What are the difficulties concerning the solution theory of such a system of equations? One way of approach is to define a self-map H on a suitable Banach space of trajectories $t \mapsto (\vec{q}_{i,t}, \vec{p}_{i,t})_{i=1,2}$ via the integral equation corresponding to (6)-(8) for prescribed initial positions and momenta $(\vec{q}_{i,0}, \vec{p}_{i,0})_{i=1,2}$. The existence of the solutions $t_{j\pm}$ to (8) is unproblematic as long as the trajectories maintain velocities smaller than one, i.e., speed of light. Nevertheless, if the velocity is a priori not bounded away from one the advanced and delayed times are potentially unbounded. Hence, there is no such thing as local solutions as the accelerations $\dot{\vec{q}}_{i,t}$ in a neighborhood of a time instant t depended on the positions $\vec{q}_{j\pm}$ at times $t_{j\pm}$. For a successful application of Banach's fixed-point theory one would need a very good a priori estimate on the behavior of trajectories $t \mapsto (\vec{q}_{i,t}, \vec{p}_{i,t})_{i=1,2}$ on potentially the whole domain \mathbb{R} , a fact that makes uniqueness results particularly difficult. Even worse, having only an existence result in the spirit of Schauder's theorem in mind, to show that H is a self-map would also require a priori bounds on the asymptotic behavior of those trajectories.

When restricting the dynamics of two repelling charges obeying (6)-(8) to a straight line, e.g., by setting $\vec{q}_{i,t} = (q_{i,t}, 0, 0)$ for $i = 1, 2$, it is fortunately possible to a priori bound their velocity away from one. The estimate is based on a comparison with the kinetic and potential energy of corresponding system without delays. It is remarkable that this bound can be given with the knowledge of the positions and momenta of both charges at initial time $t = 0$ only. From it, one can furthermore bound the minimal distance between charges. However, the estimates are too weak to show that H is a self-map. To show existence of solution one therefore falls back to the Leray-Schauder Theorem:

Theorem. *Let $(B, \|\cdot\|)$ be a Banach space, $\Omega \subset B$ a bounded open subset containing the origin and $H : [0, 1] \times \bar{\Omega} \rightarrow B$ a compact homotopy such that $H(0, \cdot)$ is the zero mapping and none of the mappings $H(\lambda, \cdot)$ for $\lambda \in [0, 1]$ has a fixed point on $\partial\Omega$. Then $H(1, \cdot)$ has a fixed point.*

To apply this theorem, one employs the Banach space of pairs of trajectories $\mathbb{R} \ni t \mapsto (\Delta q_{i,t}, \Delta p_{i,t})_{i=1,2}$, whose position and momentum vanish at

$t = 0$, equipped with the $L^\infty(\mathbb{R})$ norm on the momenta. These pairs of trajectories are to be seen as possible correction to a first guess of trajectories $t \mapsto (q_{i,t}^0, p_{i,t}^0)_{i=1,2}$ which go through prescribed initial positions and momenta at $t = 0$, and which due to linearity cannot be implemented in B directly. Regarding the position trajectory correction $\Delta q_{i,t}$, H is then defined as the difference between the auxiliary trajectories $q_{i,t}^0$ and the convex combination of $(\lambda - 1)$ times the first integral of the velocities $p_{i,t}^0 / \sqrt{(p_{i,t}^0)^2 + m_i^2}$ of the auxiliary trajectory plus λ times the twice integrated right-hand side of (7). By definition, we thus have $H[0, \cdot] = 0$ and that the fixed-points of $H[1, \cdot]$ fulfill (7). The constraints from the a priori bounds on the maximum velocities and the minimum distance of fixed-points of $H[\lambda, \cdot]$ are then exploited to define an open subset Ω of B such that all points in $\partial\Omega$ violate these constraints. Compactness and continuity can be read off the explicit form of the right-hand side of (7) again relying on the a priori estimates. As a result one infers [8]:

Theorem. *Given initial positions and momenta $(q_{i,0}, p_{i,0})_{i=1,2} \in \mathbb{R}^2$ there is at least one $C^\infty(\mathbb{R}, \mathbb{R}^2)$ solution $t \mapsto (q_{i,t}, p_{i,t})_{i=1,2}$ to (6)-(8).*

This seems like an awful lot of trouble for such a simple looking delay equation without even getting our hands on a uniqueness assertion. The toy model was therefore designed to allow for an another, easier access to potential solutions (see [9]) by what is known as "the method of steps" in the field of delay equations. Reconsidering (6) again, one realizes that the function $f(x) = x/|x|^3$ has the inverse $g(y) = y/|y|^3/2$ except for the points $x = 0$ and $y = 0$. Thus, (6) can be rewritten in terms of

$$q_{j+} = q_{i,t} + g\left(\frac{\dot{p}_{i,t}}{e_i e_j} - f(q_{j-} - q_{i,t})\right). \quad (9)$$

Note that knowing $|q_{j+} - q_{i,t}|$ also allows to compute t_{j+} according to (8). In consequence, from the knowledge of $q_{i,t}$, $\dot{p}_{i,t}$, and q_{j-} one can infer the space-time point (t_{j+}, q_{j+}) belonging to trajectory $t \mapsto q_{j,t}$. In terms of the time t_{j+} , equation (9) is a reformulation of (7) containing only delayed terms. Let us consider prescribed initial data $q_{i,t}$ for $t \in [t_i^{(0)}, t_i^{(1)}]$ and $q_{j,t}$ for $t \in (t_j^{(0)}, t_j^{(1)})$ as depicted in Figure 2. Here, $t_i^{(0)}$ is the advanced time w.r.t. $t_j^{(0)}$,

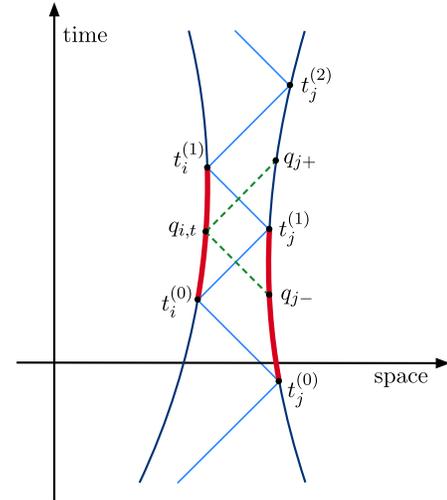


Figure 2: The initial data (in red) for the method of steps. The dashed, green line depicts the relation (9). The blue lines denote the edges of the respective light cones.

i.e., $t_i^{(0)} = t_j^{(0)} + |q_{i,t_i^{(0)}} - q_{j,t_j^{(0)}}|$, whereas $t_j^{(1)}$ is the advanced time w.r.t. $t_i^{(0)}$, and so on. From this initial data, the whole trajectory strip $q_{j,t}$ on the interval $t \in (t_j^{(1)}, t_j^{(2)})$ can be constructed exploiting (9). Once this is accomplished one can apply a similar scheme in a next step to construct the trajectory strip of $q_{i,t}$ on the interval $t \in (t_i^{(1)}, t_i^{(2)})$, and similarity one proceeds also step by step for remaining future and past trajectories strips.

However, there is a caveat: The light-cone intersection points, and hence, the advanced times $t_i^{(n)}, t_j^{(n)}$ may fail to exist starting from some $n > 1$. This is the case if the argument of the map g runs into a zero. One can provoke such situations choosing initial data disrespecting the global property of solutions to (7) that $\dot{p}_{i,t}$ and $\dot{p}_{j,t}$ must have a globally fixed and opposite sign in the case of repelling charges. If the initial data fails to respect this property, due to the repulsion of the charges, there has to be a point in the future when the argument of g in (9) approaches zero. As a result the other trajectories will be forced to asymptotically approach the speed of light in order to prevent the singularity, and the next advanced time will fail to exist.

But how can this be true when we just said that we have a global bound on the velocities depending on Cauchy data only? The subtlety to be recognized here is the difference in the notion of what we call a solution to (6)-(8). For example, above we admitted only global solutions which have to be globally smooth because of (7). The trajectories constructed by means of the method of steps above may fulfill (7) only in a piecewise manner. At each of the advanced times $t_k^{(n)}, k = 1, 2$, the left-hand and right-hand side derivatives may differ. The regularity of solutions constructed by the methods of steps can be increased by requiring that the initial data ought to fulfill (7), including any derivatives of it, at $t = t_i^{(0)}$, and analogously for the corresponding equation for charge j at $t = t_j^{(1)}$. However, the exact requirements needed to infer strong global solutions to (6)-(8) are unknown. So far we can only infer the following:

Theorem. *Given a strong global solution $\mathbb{R} \ni t \mapsto (q_{i,t}, p_{i,t})_{i=1,2}$ to (6)-(8), the trajectory strip given by $q_{i,t}$ for $t \in [t_i^{(0)}, t_i^{(1)}]$ and $q_{j,t}$ for $t \in [t_j^{(0)}, t_j^{(1)}]$ is sufficient data to reconstruct the solution uniquely.*

Beyond that the only known solutions to (6)-(8) are explicit two-dimensional periodic orbits, i.e., charges revolving around each other in a plane with constant angular speed [9].

Outlook

Many features of the toy model carry over to the true FST equations (1). For them, e.g., the periodic orbits are known as Schild solutions [12]. That they exhibit no radiative energy loss fits well with the existence of a conserved energy and momentum functionals for FST solutions [16]. Unfortunately, it is unclear how to exploit these functionals to infer a priori bounds on the velocities given Cauchy data only in order to study the corresponding map H as above. In fact, these functionals depend on the initial data shown in Figure 2. In [8], where we discuss existence of FST solutions on the future half-space, we preliminarily circumvent this problem by prescribing initial strips while looking for trajectories that extend the initial strips and satisfy the FST equations, at least outside the initial strips. The 3+1 dimensional FST equations exhibit the additional difficulty of being neutral. One promising line of attack is the reformulation of the FST equation by means of the Maxwell-Lorentz equations (3)-(4) described above. In [6] this idea is implemented in the case of N smeared-out charges in 3+1 dimensions. There, existence of conditional FST solutions is shown, i.e., trajectories that, for arbitrary large $T < \infty$, solve the FST equations on the interval $(-T, T)$ while they are extended by asymptotes of prescribed shape for times $|t| \geq T$. Both, there and in [8], the lack of a priori bounds prevents obtaining global solutions by a limiting procedure. In turn, in [2] existence of global solutions restricted on the straight line for prescribed asymptotic behavior is obtained, leaving open their connection to finite time empirical data and the question of uniqueness. Another line of attack is to employ the variation problem above directly to find conditional solutions [3].

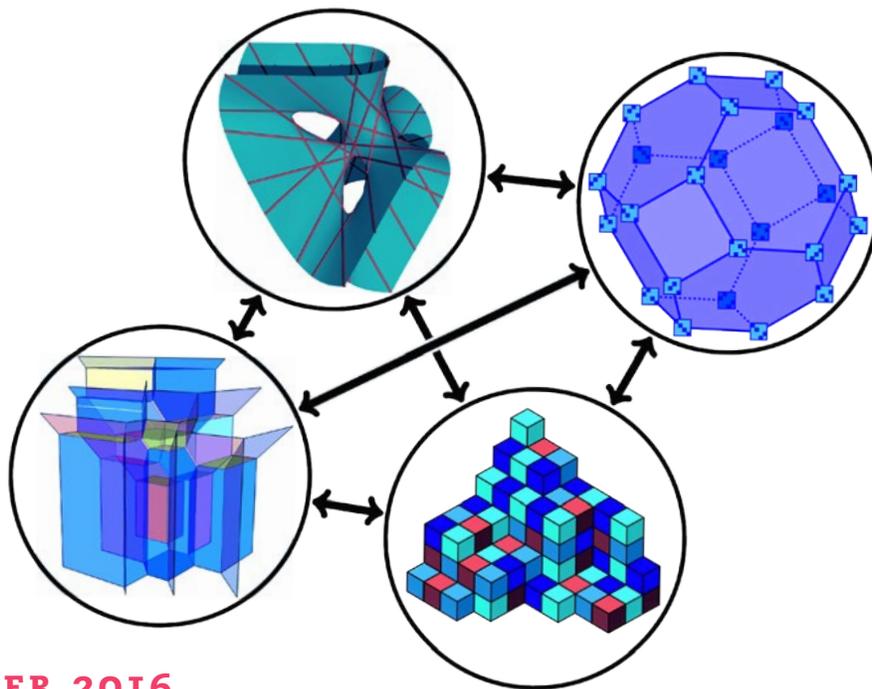
At present, it is unclear what sort of data – Cauchy data, trajectory strips, etc. – is necessary to determine solutions uniquely. If it were strictly strips as in Figure 2, maybe even without lots of additional conditions, the problem of classifying solutions would still break down in the case of thermodynamically large systems of particles as the light-cone structure of the end points of such strips would essentially require to include the entire trajecto-

ries. Such large systems are needed for Wheeler and Feynman's derivation of the radiation reaction equation. Nevertheless, there is well-founded hope that the situation is not as bad. An early indicating that is also [10], stating that at least two like charges moving symmetrically on a line sufficiently far apart have unique solutions classified by Cauchy data given at their turning point.

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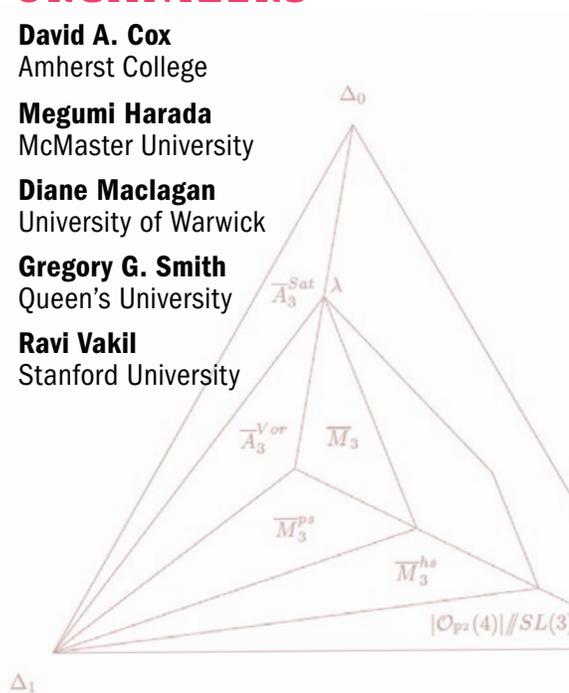
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October 19-22, 2015

Fields Medal Symposium in honour of Stanislav Smirnov
Fields Institute

THEMATIC AND FOCUS PROGRAMS 2015 – 2016

January - June 2015

Thematic Program on Statistical Inference, Learning and Models for Big Data

July - December 2015

Thematic Program on Computer Algebra

January - April 2016

Thematic Program on Multiscale Scientific Computing: from Quantum Physics
and Chemistry to Material Science and Fluid Mechanics

May - June 2016

Focus Program on Nonlocal Partial Differential Equations

August 2016

Focus Program on Topology, Stratified Space and Particle Physics

July - December 2016

Thematic Program on Combinatorial Algebraic Geometry

For more information about our activities, please visit
www.fields.utoronto.ca/programs

CALL FOR PROPOSALS, NOMINATIONS, AND APPLICATIONS

For more information about Fields Institute proposals, nominations, or applications, please visit our website: www.fields.utoronto.ca/proposals

THEMATIC AND FOCUS PROGRAMS

The Fields Institute solicits proposals for a variety of programs in areas of current research interest in the mathematical sciences: (1) Major thematic programs, six months in length. (2) Thematic or focus programs, from one to two months in length to run concurrently with our major thematic programs; in particular, two-month summer programs of an interdisciplinary nature. Proposals or letters of intent should be submitted by March 15 or September 15, with a lead time of at least two years recommended for six-month programs.

GENERAL SCIENTIFIC ACTIVITIES

Proposals for short scientific events in the mathematical sciences should be submitted by October 15, February 15, or June 15 of each year, with a lead time of at least one year recommended. Activities supported include workshops, conferences, seminars, and summer schools.

OUTREACH PROPOSALS

The Fields Institute provides support for projects whose goal is to promote mathematical culture at all levels and bring mathematics to a wider audience. Faculty at Fields sponsoring universities or affiliates are invited to submit a proposal to the Fields Outreach Competition. There are two submission deadlines each year, June 1 and December 1. Proposals should include a detailed description of the proposed activity and the target audience. A budget indicating other sources of support is also required.

FIELDS INSTITUTE FELLOWS

This is a call for nominations for the Fields Institute Fellows. To nominate someone, please send a CV plus a letter briefly outlining why your candidate is a worthy nominee, to:

The Director
Fields Institute
222 College Street, Second Floor
Toronto, Ontario M5T 3J1, Canada

or to proposals@fields.utoronto.ca

The annual deadline is February 15.

Winners of the CRM-Fields-PIMS prize are automatically recommended for fellowship. No member of the current Fields Institute Board of Directors nor any continuing member of the Fields Institute Scientific Advisory Panel will be eligible. Nominations are encouraged from all qualified individuals, including women, members of visible minorities, and persons with disabilities.

THE DEAN'S DISTINGUISHED VISITING PROFESSORSHIP

The Dean's Distinguished Visiting Professorship is a joint program of the Fields Institute with the Faculty of Arts and Science, and the Department of Mathematics of the University of Toronto. Each year, the program brings a leading international researcher in the mathematical sciences to give a full-term course connected to a Fields Institute program, for graduate and advanced undergraduate students of the University of Toronto and other students participating in the program.

The Dean's Distinguished Visiting Professorship currently provides a stipend of \$50,000, for a visitor to be in residence throughout a term. The Dean's Distinguished Visiting Professor is selected by a committee representing the Fields Institute and the Department of Mathematics. Nominations can be made either to the Director of the Institute or to the Chair of the Department of Mathematics.

THE MARGARET SINCLAIR MEMORIAL

The Margaret Sinclair Memorial Award recognizes an educator in Canada who has demonstrated innovation and excellence in promoting mathematics education at the elementary, secondary, college or university level. This annual award will be administered by the Fields Institute for Research in Mathematical Sciences and comprises a \$5000 prize and inscription of the winner's name on a plaque to be installed at the Fields Institute recognizing the recipients. Candidates for the award will have shown an enthusiasm for enhancing the learning environment and displaying novel ideas, methods or devices for teaching mathematics. They will also have supporting evidence of providing an opportunity for students to achieve, observe, and experience mathematics in a thoughtful and significant way.

Creative leadership is also an important factor. Candidates must have served as a role model for other teachers and had an inspirational influence on students and/or colleagues. This may have been displayed through meaningful classroom teaching, directing mathematical forums, encouraging rational and critical discussions while using accurate data to support one's position, fostering teamwork, writing effectual curriculum support materials, contributing to mathematics education journals, or being involved as an organizer or speaker at mathematics education meetings or conferences.

Candidates for the award may nominate themselves or be nominated by others.

A complete nomination packages consists of:

1. A nomination letter of no more than 1000 words demonstrating alignment with The Margaret Sinclair Memorial Award
2. A recent curriculum vitae
3. Three (3) letters from arm's length referees

Nominations for the 2016 Margaret Sinclair Memorial Award must be received electronically by the Fields Institute by December 1, 2015. Send to deputydirector@fields.utoronto.ca

Please note that nominations for the Margaret Sinclair Award will be kept under consideration for two additional years following the initial submission.

POSTDOCTORAL FELLOWSHIPS

The Fields Institute's Postdoctoral Fellowships provide for a period of research activity at the Institute and participation in our programs. We are currently soliciting applications for Fields Postdoctoral Fellowships and Jerrold E. Marsden Postdoctoral Fellowships. Qualified candidates who will have a recent PhD (normally awarded not more than five years before tenure of the Fellowship) are encouraged to apply.

CALL FOR PROPOSALS, NOMINATIONS, AND APPLICATIONS *CONTINUED...*

FIELDS RESEARCH IMMERSION FELLOWSHIP

This program supports individuals with high potential to re-enter an active research career after a career interruption. The RIF is expected to participate fully in one or more of the thematic programs, in the expectation that this will allow her or him to enhance her or his research capabilities and to establish or re-establish a career as a productive, competitive researcher.

For programs in a given program year (which runs July to June) the closing date will be the preceding March 31, but late applications will be considered if the position has not yet been filled.

Please submit a cover sheet indicating your interests, a CV, a brief description of the circumstances altering your research activity, and a proposal describing the impact you expect a semester at Fields to have on your research career. Include the names of faculty who will be participating in the Thematic Program who may be appropriate as supervisors or research advisors.

Application should be sent by email to:

Director : director@fields.utoronto.ca

Fields Institute for Research in Mathematical Sciences

FIELDS-PERIMETER AFRICA POSTDOCTORAL FELLOWSHIP

The Fields Institute for Research in Mathematical Sciences and Perimeter Institute for Theoretical Physics are inviting applications from African Nationals for a one-year Joint Postdoctoral Fellowship. The deadline for applications to the 2016 Fields-Perimeter Africa Postdoctoral Fellowship will be November 15, 2015.

Applications are accepted through Mathjobs:

<https://www.mathjobs.org/jobs/jobs/5691>

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THEMATIC PROGRAM ON COMPUTER ALGEBRA

JULY - DECEMBER, 2015 | THE FIELDS INSTITUTE

SEPTEMBER 14 - 19, 2015

**WORKSHOP ON SYMBOLIC
COMBINATORICS AND
COMPUTATIONAL DIFFERENTIAL
ALGEBRA**

OCTOBER 26 - 31, 2015

**WORKSHOP ON LINEAR
COMPUTER ALGEBRA
AND SYMBOLIC-NUMERIC
COMPUTATION**

DECEMBER 7 - 16, 2015

**WORKSHOP ON ALGEBRA,
GEOMETRY AND PROOFS IN
SYMBOLIC COMPUTATION**

COXETER LECTURE SERIES

VICTOR SHOUP, COURANT INSTITUTE

COMPUTER ALGEBRA DAYS

ORCCA SESSIONS

Four full-day "Computer Algebra Days" will be held, open to the Thematic Program participants, the Ontario computer algebra community and other interested parties.

The **Thematic Program on Computer Algebra** focuses on current and emerging areas of importance in computer algebra: symbolic linear algebra, algorithms for algebraic and semi-algebraic geometry, hybrid symbolic-numeric computing, algorithms for differential algebra and symbolic combinatorics, and the area of computer-enhanced mathematical collaboration and certified algorithms.

ORGANIZING COMMITTEE

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PROGRAM-SPECIFIC COURSES

Overview of Computer Algebra Graduate Course

Research Topics of Computer Algebra Graduate Course

For more information, please visit:

<http://www.fields.utoronto.ca/programs/scientific/15-16/computeralgebra>

